Relativistic stellar aberration for the Space Interferometry Mission (2)

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ABSTRACT

We address the issue of relativistic stellar aberration requirements for the Space Interferometry Mission (SIM). Motivated by the importance of this issue for SIM, we have considered a problem of relative astrometric observations of two stars separated by angle θ on the sky with a single baseline interferometer. While a definitive answer on the stellar aberration issue may be obtained only in numerical simulations based on the accurate astrometric model of the instrument, one could still derive realistic conclusions by accounting for the main expected properties of SIM. In particular, we have analysied how the expected astrometric accuracy of determination of positions, parallaxes and proper motions will constrain the accuracy of the spaceraft navigation. We estimated the astrometric errors introduced by imperfect metrology (variations of the calibration term across the tile of interest), errors in the baseline length estimations, and those due to orbital motion of the spacecraft. We also estimate requirements on the data sampling rate necessary to apply on-board in order to correct for the stellar aberration. We have shown that the worst case observation scenario is realized for the motion of the spacecraft in the direction perpendicular to the tile. This case of motion will provide the most stringent requirement on the accuracy of knowledge of the velocity's magnitude. We discuss the implication of the results obtained for the future mission analysis.

Subject headings: astrometry; techniques: interferometric, SIM; methods: analytical; solar system; relativity

Introduction

One of the best promising methods of searching for a planetary systems around nearby stars is the high accuracy astrometric observations. SIM is being designed to produce wealth of the astrometric data necessary to address exactly this problem. It is clear that the

corresponding astrometric signal due to a reflex motion of a target star will be at the level of a few μas and smaller. The most successful searching techniques will have to incorporate an intelligent way of data processing almost at the astrometric noise level. Another important part of this puzzle is development of the astrometric model for the instrument. This model should include all the parameters necessary to account for different phenomena affecting the light propagation, and are due to the interstellar media, the solar system dynamics, as well as due to the motion of the free-flying interferometer itself. As a result, the accuracy of astrometric observations expected with SIM, will require a number of dynamical parameters to be precisely known. To do this, one will have to be able to remove (or to correctly account for) the signatures of all the known effects in order to study the unknown phenomena. This is necessary for minimizing probability of a false detections caused, for example, by aliasing the dynamics of objects in the solar system. Actually, there will be a number of parameters introduced by the dynamics in the solar system with periods of the order of a few years. One of such a parameters is the three-dimensional vector of barycentric velocity of the spacecraft. Thus, a future astrometric model will have to account not only for the effects due to the motion of the solar system bodies, but also for those that are generated by the motion of the spacecraft and corresponding errors in the spacecraft's navigation.

In this paper we will discuss some important elements of dynamical model for the future astrometric observations with SIM. Specifically, we will address the issue of the relativistic stellar aberration. The stellar aberration is a very important problem for SIM. Effect of the barycentric velocity of the spacecraft is the largest term in the SIM astrometric model and will amounts to ~ 20.5 arcsec for the Earth-trailing orbit. This effect is important not only for a wide angle astrometric observations, but it will produce a measurable effect for a pairs of widely separated stars even inside the tile (with diameter $\sim 15^\circ$). Additionally, a possible correlation of the astrometric observables with the errors in the spacecraft's velocity sky-angles suggests that in order to achieve the mission accuracy for a global astrometric observations of $\sigma = 4$ μas, one will have to control the barycentric velocity of the spacecraft throughout the entire mission and account for the relativistic stellar aberration inside every single tile.

The outline of this paper is as follows. In Section 1 we will present a simplified model for differential astrometric observations with a free-flying interferometer with a single baseline. We will discuss a model for absolute and differential astrometric measurements and will give a three-dimensional parameterization of main observables. We will present the first order differentials, necessary to analyze the errors propagation. In Section 2 we will discuss orientation of the tile in the SIM nominal observing direction. We will present the first order equations that are governing the propagation of astrometric errors in the instrument. Specifically, we address the velocity knowledge requirement, the data sampling rate and orbital

position knowledge, and the spacecraft acceleration accuracy constraint. In Section 3 we will derive the set of the relativistic stellar aberration requirements for SIM. We will conclude by presenting our recommendations for future attitude control for the SIM spacecraft. In order to make access of the basic results of this paper easier, we will present some important (but lengthy!) calculations in the Appendices.

1. A model for the relativistic stellar aberration

In this Section we will derive a model necessary to analyze the astrometric error budget for differential astrometric observations with a single-baseline interferometer. To do this, we will present estimates of the optical path difference (OPD) for a single-baseline interferometer in solar orbit. Our derivations will be different from the ones obtained earlier by the fact that for each tile we account for effects of general three-dimensional motion of the interferometer, for the errors in the components of the baseline vector $\vec{b}$, and for the error in estimating the instrumental offset (or, calibration) term, c_0. Our model is very simple, but it could be easily expanded to accommodate a number of other important features of the instrument.

We address the problem of relativistic stellar aberration from a general standpoint. A "toy" astrometric model developed here will include only the largest relativistic contributions due to the orbital motion of the interferometer around the sun. A complete, fully-relativistic model for the SIM observations, will have to account for a number of physical phenomena and should include a set of additional terms (and corresponding parameters) necessary for different astrometric applications. More specifically, the future model should include the terms $\propto (\frac{v}{c})^3$ to account for higher orders of relativistic stellar aberration; terms $\propto\ G\mathcal{M}/c^2$ (where G is the gravity constant and $\mathcal{M}$ is the mass of the deflecting body) to account for the gravitational deflection by the bodies of the solar system (primarily the Sun and the Jupiter). (For general discussion of the general relativistic effects in the SIM asdtrometric observations, see Turyshev (2002)) One also will need to account for rigid-body rotational motion of the spacecraft, etc. All these effects, together with a number of others, are out of the scope of the present paper, but will be discussed elsewhere. While the derivation of the general expressions will be given in the reminder of the document, here we will present only the main results obtained.

1.1. Geometry of the problem

Let us begin by deriving the expression for OPD ℓ for an interferometer with a single baseline $\vec{b}$ which is moving with the velocity $v \ll c$ (for SIM this ratio will of of the order $v/c \sim 10^{-4}$) with respect to some reference frame (RF), for example the solar-system barycentric (SSB) reference frame. In the interferometer's proper quasi-inertial reference frame OPD, ℓ', may be given as follows:

$$\ell' = (\vec{b'} \cdot \vec{s'}) + c'_0, \tag{1}$$

where $\vec{b'}$ is the baseline vector, $\vec{s'}$ is the direction to the observed source, and the last term in this expression is the instrument offset (or "constant" term), c_0, that can be calibrated out. The same quantity may be presented in the coordinates of the SSB reference frame. The corresponding transformation between the solar system barycentric frame and the one of the moving interferometer (with velocity $\vec{v}$ with respect to SSB) will necessarily account for the effect of the relativistic stellar aberration (for more details see Turyshev (1998)). To the first order in $\frac{v}{c}$, the aberrated direction to the source and the baseline vector may be expressed as follows:

$$\vec{s'} = \vec{s} + \frac{\vec{v}}{c} - \frac{\vec{s}(\vec{s}\cdot\vec{v})}{c} + \mathcal{O}\Big(G, (\frac{v}{c})^2\Big), \tag{2}$$

$$\vec{b'} = \vec{b} + \mathcal{O}\Big(G, (\frac{v}{c})^2\Big), \tag{3}$$

where $\vec{s}$ and $\vec{b}$ are the direction to the observed source and the baseline vector as measured in SSB. The total effect of this transformation has the form

$$\ell = (\vec{b}\cdot\vec{s})\Big(1 - \frac{(\vec{s}\cdot\vec{v})}{c}\Big) + \frac{(\vec{b}\cdot\vec{v})}{c} + c_0 + \mathcal{O}\Big(G, (\frac{v}{c})^2\Big), \tag{4}$$

where we have assumed that the instrument offset does not depend on the velocity of the spacecraft motion to the second order in $\frac{v}{c}$, or, in more general terms, $c'_0 = c_0 + \mathcal{O}\Big(G, (\frac{v}{c})^2\Big)$. This issue may be correctly addressed later when the accurate model of the instrument will be developed.

The simplified two-dimensional geometry of the problem presented in Figure 1. This allows to express the scalar products in Eq.(4) and re-write this equation in a more familiar form

$$\ell = b\cos\alpha\Big(1 - \frac{v}{c}\cos(\alpha - \psi)\Big) + \frac{bv}{c}\cos\psi + c_0. \tag{5}$$

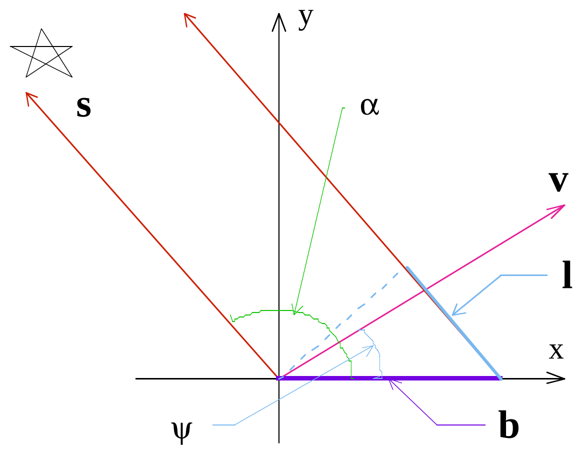

Fig. 1.— Two-dimensional geometry of the problem: absolute astrometry.

Such an expression may be used to model absolute astrometric observations with a moving interferometer, when the observed sources are not in the same field of view, but rather widely separated from each other. However, the wide-angle astrometric campaign with SIM will be based on a set of differential (or relative) astrometric observations made inside a set of 637 tiles covering the whole sky (Boden (1997); Swartz (1998)). As a result, it will measure to a certain accuracy the angular distances between a pairs of stars in the same field of view. This necessitates the derivation of an expression for the optical path difference, similar to Eq.(5), which would take this fact into account. To the first order in $\frac{v}{c}$ this is a fairly easy task. Assuming that absolute position for each star is given by expression, similar to that of Eq.(4), namely:

$$\begin{aligned} \ell_1 &= (\vec{b}\cdot\vec{s}_1)\Big(1-\frac{(\vec{s}_1\cdot\vec{v})}{c}\Big)+\frac{(\vec{b}\cdot\vec{v})}{c}+c_{01}, \\ \ell_2 &= (\vec{b}\cdot\vec{s}_2)\Big(1-\frac{(\vec{s}_2\cdot\vec{v})}{c}\Big)+\frac{(\vec{b}\cdot\vec{v})}{c}+c_{02}, \end{aligned} \tag{6}$$

one could write the corresponding differential (or relative) OPD for a differential astrometric observations. To describe differential measurements one will have to subtract one OPD from the other, namely $\delta\ell = \ell_1 - \ell_2$. The corresponding differential OPD $\delta\ell$, will be given as follows:

$$\delta\ell = (\vec{b}\cdot\vec{s}_1)\Big(1-\frac{(\vec{s}_1\cdot\vec{v})}{c}\Big)-(\vec{b}\cdot\vec{s}_2)\Big(1-\frac{(\vec{s}_2\cdot\vec{v})}{c}\Big)+\delta c_0+\mathcal{O}\Big(G,(\frac{v}{c})^2\Big), \tag{7}$$

where δc_0 is the differential “constant” term $\delta c_0 = c_{01} - c_{02}$.

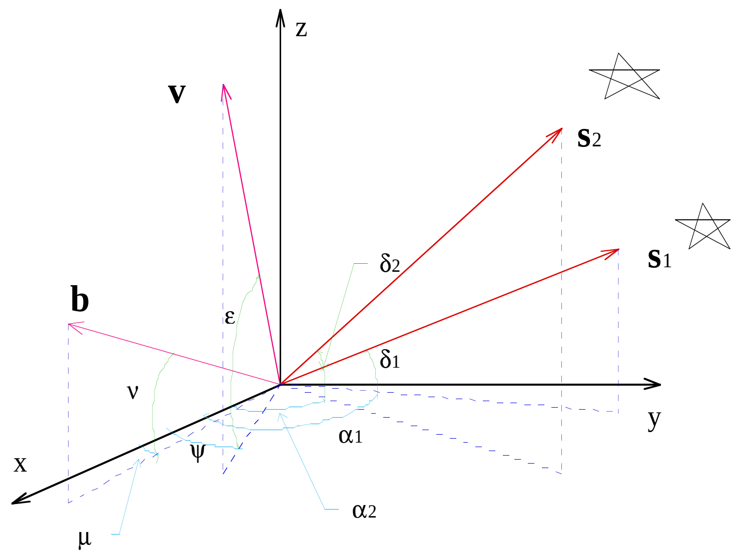

Fig. 2.— Geometry of the problem: differential astrometry.

1.2. Three-dimensional parameterization.

In this subsection we will present the three-dimensional parameterization of all the vectors involved in the problem. This from of representation is more useful for practical applications, for example, to study the orbital motion of the spacecraft; for analyzing the various requirements on the spacecraft's velocity, rigid-body rotations, vibrations of the whole structure, etc.

The overall three-dimensional geometry of the problem presented in Figure 2. The different vectors involved are given in the spherical coordinate system by their magnitudes and two corresponding sky-angles:

$$\begin{aligned}
\vec{s}_1 &= \big(\cos\alpha_1\cos\delta_1,\ \ \sin\alpha_1\cos\delta_1,\ \ \sin\delta_1\big),\\
\vec{s}_2 &= \big(\cos\alpha_2\cos\delta_2,\ \ \sin\alpha_2\cos\delta_2,\ \ \sin\delta_2\big),\\
\vec{v} &= v\big(\cos\psi\cos\epsilon,\ \ \sin\psi\cos\epsilon,\ \ \sin\epsilon\,\big),\\
\vec{b} &= b\big(\cos\mu\cos\nu,\ \ \sin\mu\cos\nu,\ \ \sin\nu\,\big).
\end{aligned} \tag{8}$$

This parameterization allows one to present expression Eq.(7) in the following form:

$$
\begin{aligned}
\delta\ell &= b\Big[\cos\nu\Big(\cos\delta_1\cos(\alpha_1-\mu)-\cos\delta_2\cos(\alpha_2-\mu)\Big)+\sin\nu\Big(\sin\delta_1-\sin\delta_2\Big)\Big]+\delta c_0- \\
&- \frac{bv}{c}\Big[\Big(\cos\nu\cos\delta_1\cos(\alpha_1-\mu)+\sin\nu\sin\delta_1\Big)\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big)- \\
&- \Big(\cos\nu\cos\delta_2\cos(\alpha_2-\mu)+\sin\nu\sin\delta_2\Big)\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big)\Big]. \qquad (9)
\end{aligned}
$$

This expression is quite difficult for analytical description, however, for the purposes of the present study, it may be simplified (a more detailed analysis of this problem will be given in the Appendix A). Thus, one may see that the angles of the baseline orientation (μ, ν) are significantly influencing the narrow-angle astrometric observations. Angle μ corresponds to a shift of the origin of `RA` for the <u>*tile*</u> under consideration and, without lost of generality, it may be omitted (e.q. set to be zero). The analysis of contribution of the `DEC` angle ν is a little bit more complicated task. For the purposes of this paper we will assume that this angle will be $\nu = 0$ at the beginning of the <u>*tile*</u> observation and will insignificantly deviate from this value during the experiment. This is true, because one of the functions for the two guide interferometers in SIM will be to provide a stable reference for the science interferometer. We, therefore will set both angles as $\mu = \nu = 0$. These parameters will have to be incorporated in the future astrometric model for SIM, probably in a form of the rigid body rotation/precession/nutation model of the spacecraft. At this moment, this choice is equivalent to choosing the direction of the baseline vector $\vec{b}$ coinciding with x-axis: $\vec{b} = b(1, 0, 0)$ (see Figure 2). As a result, all vectors now will be counted <u>from the baseline</u> and the expression (9) may now be presented in a simpler form, namely:

$$
\begin{aligned}
\delta\ell &= b\Big(\cos\delta_1\cos\alpha_1-\cos\delta_2\cos\alpha_2\Big)+\delta c_0- \\
&- \frac{bv}{c}\Big[\cos\delta_1\cos\alpha_1\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big)- \\
&\qquad -\cos\delta_2\cos\alpha_2\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big)\Big]. \qquad (10)
\end{aligned}
$$

The obtained expression may now be used to analyze the error propagation for the moving interferometer. To do this, one will have to expand this expression in a Taylor series and keep only the first term of the expansion. While the details of this calculations are given in Appendix A, here we present only the final expression which is necessary to analyze contributions of the chosen set of different error factors to the overall error budget:

$$
\begin{aligned}
\frac{\Delta\delta\ell}{b} &= \Delta\alpha\, \sin\alpha_2\cos\delta_2 + \Delta\alpha_1\Big(\sin\alpha_2\cos\delta_2 - \sin\alpha_1\cos\delta_1\Big) + \\
&+ \Delta\delta\, \cos\alpha_2\sin\delta_2 + \Delta\delta_1\Big(\cos\alpha_2\sin\delta_2 - \cos\alpha_1\sin\delta_1\Big) + \\
&+ \frac{\Delta b}{b}\Big(\cos\alpha_1\cos\delta_1 - \cos\alpha_2\cos\delta_2\Big) + \frac{\Delta\delta c_0}{b} + \\
&+ \Big[\frac{\Delta v}{c}\cos(\epsilon-\epsilon_0) - \Delta\epsilon\, \frac{v}{c}\sin(\epsilon-\epsilon_0)\Big]\sqrt{(a^2+f^2)\cos^2(\psi-\psi_0)+k^2} - \\
&- \Delta\psi\, \frac{v}{c}\cos\epsilon\sqrt{a^2+f^2}\,\sin(\psi-\psi_0), \qquad (11)
\end{aligned}
$$

where we have used the following definitions for the situation discussed:

$\alpha = \alpha_2 - \alpha_1$	:	`RA` angle between the two stars *inside* the tile under consideration;
$\delta = \delta_2 - \delta_1$	:	`DEC` angle between the two stars *inside* the tile under consideration. [Note that the following identity is expected to hold for these two angles $\Rightarrow$ $\alpha^2 + \delta^2 \le (\frac{\pi}{12})^2$];
$\Delta\alpha$	:	error in defining α: $\Delta\alpha = \alpha_{\tt true} - \alpha_{\tt estimated}$;
$\Delta\delta$	:	error in defining δ: $\Delta\delta = \delta_{\tt true} - \delta_{\tt estimated}$;
$\Delta\alpha_1$	:	error in defining `RA` angle, α_1, for the primary star $\Delta\alpha_1 = \alpha_{1\tt true} - \alpha_{1\tt estimated}$;
$\Delta\delta_1$	:	error in defining `DEC` angle, δ_1, for the primary star $\Delta\delta_1 = \delta_{1\tt true} - \delta_{1\tt estimated}$;
Δb	:	error in defining the baseline length b: $\Delta b = b_{\tt true} - b_{\tt estimated}$;
$\Delta\delta_{c_0}$	:	error in defining the relative constant term for the observations *inside* the tile: $\Delta\delta_{c_0} = \delta_{c_0\tt true} - \delta_{c_0\tt estimated}$;
$\Delta v,\ \Delta\epsilon,\ \Delta\psi$	:	errors in knowledge of components of the three-dimensional spacecraft velocity vector, defined as usual $\Rightarrow$ $\Delta =$ `true - estimated`.

Additionally, the new quantities ψ_0 and ϵ_0 are entirely defined by the coordinates of the two stars under consideration and are given as follows:

$$\begin{aligned}
\sin\psi_0 &= \frac{f}{\sqrt{a^2+f^2}}, \qquad \cos\psi_0 = \frac{a}{\sqrt{a^2+f^2}},\\
a &= \cos^2\delta_2\cos^2\alpha_2 - \cos^2\delta_1\cos^2\alpha_1,\\
f &= \cos^2\delta_2\cos\alpha_2\sin\alpha_2 - \cos^2\delta_1\cos\alpha_1\sin\alpha_1. \qquad (12)\\
\sin\epsilon_0 &= \frac{k}{\sqrt{(a^2+f^2)\cos^2(\psi-\psi_0)+k^2}},\\
\cos\epsilon_0 &= \frac{\sqrt{a^2+f^2}\cos(\psi-\psi_0)}{\sqrt{(a^2+f^2)\cos^2(\psi-\psi_0)+k^2}},\\
k &= \cos\delta_2\sin\delta_2\cos\alpha_2 - \cos\delta_1\sin\delta_1\cos\alpha_1. \qquad (13)
\end{aligned}$$

These expressions (11)-(13) may now be used to study propagation of astrometric errors to the first order. With these results one may analyze the relativistic stellar aberration issue analytically for a different positions of the two stars of interest. Note that in reality all the quantities involved are both dynamical and stochastic functions of time. This fact we be used in the Section 3 when we will analyze the allocations adopted for different constituents of the SIM astrometric error budget.

2. Tile in the SIM nominal observing direction

Analysis of the tolerable errors for the velocity aberration in a general case is a complicated problem and should be attacked by using a formal numerical treatment. However, we may simplify the task by analyzing a number of a special cases which are expected to provide the most stringent requirements on the velocity magnitude. Thus, one may expect that the most driving requirement will come in the case when the two stars are aligned in a tile in a such a way that the angular separations between them is given by $\alpha = \frac{\pi}{12}$, $\delta = 0$. It turns out that this is indeed the case. The compensation for aberration across the field of view of the interferometer in a direction parallel to the baseline, will present the greatest challenge for SIM. To demonstrate this, we will discuss a more general situation (that includes the case mentioned above), which allows one to obtain an analytical expressions useful for the future analysis.

Let us define a tile in the SIM nominal observing direction — perpendicular to the baseline. For the purposes of our analysis we will assume that positions of the two stars are symmetric with respect to the point with coordinates ($\alpha_0 = \frac{\pi}{2}, \delta_0 = 0$). Then the most

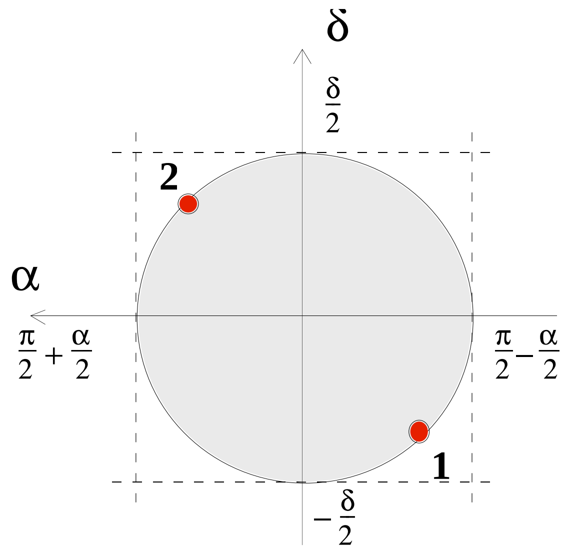

Fig. 3.— A tile in the SIM nominal observing direction. Note that $\alpha^2+\delta^2 \leq (\frac{\pi}{12})^2$.

suitable description for the nominal SIM observing mode is given by coordinates of the primary and the secondary stars as follows:

$$\begin{aligned} \alpha_1 &= \frac{\pi}{2} - \frac{\alpha}{2}, \qquad & \alpha_2 &= \frac{\pi}{2} + \frac{\alpha}{2}, \\ \delta_1 &= -\frac{\delta}{2}, \qquad & \delta_2 &= \frac{\delta}{2}. \end{aligned} \tag{14}$$

Now we can define a tile for this observing direction as a set of all the points for which the angular separations α and δ are limited as: $\alpha^2+\delta^2 \leq (\frac{\pi}{12})^2$. The resulted area constitutes the tile in the SIM nominal observing direction and it is shown as a shaded area in the Figure 3.

By substituting the coordinates for the two stars Eq.(14) in the expressions (12)-(13) we obtain the following values for the constants involved:

$$\begin{aligned} a &= 0, \\ f &= -\sin\alpha\cos^2\tfrac{\delta}{2}, \\ k &= 0, \\ a^2+f^2 &= \sin^2\alpha\cos^4\tfrac{\delta}{2}, \\ a^2+f^2+k^2 &= \sin^2\alpha\cos^4\tfrac{\delta}{2}. \end{aligned} \quad \Rightarrow \quad \begin{array}{l} \left.\begin{aligned} \sin\psi_0 &= -1 \\ \cos\psi_0 &= 0 \end{aligned}\right\} \Rightarrow \quad \psi_0 = -\frac{\pi}{2}, \\[2ex] \left.\begin{aligned} \sin\epsilon_0 &= 0 \\ \cos\epsilon_0 &= 1 \end{aligned}\right\} \Rightarrow \quad \epsilon_0 = 0. \end{array}$$

Utilizing these results in the equation (11), we obtain the expression necessary to analyze contributions of different error factors to the total differential OPD. This expression has the following form:

$$\begin{aligned}\frac{\Delta\delta\ell}{b} \;&=\; \Delta\alpha\,\cos\frac{\alpha}{2}\cos\frac{\delta}{2} \;-\; \Delta\delta\,\sin\frac{\alpha}{2}\sin\frac{\delta}{2} \;+\; \frac{2\Delta b}{b}\,\sin\frac{\alpha}{2}\cos\frac{\delta}{2} \;+\; \frac{\Delta\delta c_0}{b} \;- \\ &-\; \Big[\Big(\frac{\Delta v}{c}\,\cos\epsilon \;-\; \Delta\epsilon\,\frac{v}{c}\,\sin\epsilon\Big)\sin\psi \;+\; \Delta\psi\,\frac{v}{c}\,\cos\epsilon\cos\psi\Big]\sin\alpha\cos^2\frac{\delta}{2}. \qquad (15)\end{aligned}$$

The obtained expression Eq.(15) may now be used to analyze the propagation of errors in the future astrometric observations with SIM.

2.1. Propagation of astrometric errors

In this section we will concentrate on obtaining the expressions that are essential for derivations of the requirements on the quality of the spacecraft navigation data. These requirements are imposed by the expected accuracy of the future SIM astrometric measurements, such as determination of positions, parallaxes and proper motions.

In order to derive the necessary equations, the first term in the expression (15) $\Delta\delta\ell$ may equivalently be presented as $\Delta\delta\ell = \Delta(n\lambda_0) = \Delta n\,\lambda_0 + n\,\Delta\lambda_0$, where λ_0 is the operating frequency and n is an integer number. This term vanishes because both λ_0 and n are assumed to be known to a sufficiently high accuracy, thus $\Delta n = 0,\ \Delta\lambda_0 = 0$. This is true because SIM will be using it's fringe tracker to find a white light fringe to perform the calibration of the instrument. Then the remaining differentials $\Delta v,\ \Delta\epsilon,\ \Delta\psi,\ \Delta\alpha,\ \Delta\delta,\ \Delta b$ and $\Delta\delta c_0$ will have to satisfy the equation:

$$\begin{aligned}\Delta\alpha\,\cos\frac{\alpha}{2}\cos\frac{\delta}{2} \;-\; \Delta\delta\,\sin\frac{\alpha}{2}\sin\frac{\delta}{2} \;&=\; -\,\frac{\Delta\delta c_0}{b} - \frac{2\Delta b}{b}\,\sin\frac{\alpha}{2}\cos\frac{\delta}{2} + \\ &+\Big[\Big(\frac{\Delta v}{c}\,\cos\epsilon \;-\; \Delta\epsilon\,\frac{v}{c}\,\sin\epsilon\Big)\sin\psi \;+\; \Delta\psi\,\frac{v}{c}\,\cos\epsilon\cos\psi\Big]\sin\alpha\cos^2\frac{\delta}{2}. \qquad (16)\end{aligned}$$

In order to simplify further analysis we have separated the terms in the expression above in a such a way, so that the left side of this equation represents the error in the measurement of absolute angular separation between the two stars on the sky, while the left side shows the main contributing factors to this quantity.

2.1.1. Velocity knowledge requirement

One may expect that, in any given tile, the errors $\Delta\alpha$ and $\Delta\delta$ are normally distributed and uncorrelated.[1] Also, the errors due to the orbital dynamics $\Delta v,\ \Delta\epsilon,\ \Delta\psi$ at the chosen approximation may be treated as being uncorrelated with the instrumental errors $\Delta b,\ \Delta c_0$.[2] Then, one obtains:

$$\begin{aligned}\sigma^2_\alpha \cos^2\frac{\alpha}{2}\cos^2\frac{\delta}{2}+\sigma^2_\delta \sin^2\frac{\alpha}{2}\sin^2\frac{\delta}{2} &= \frac{\sigma^2_{\delta_{c_0}}}{b^2}+\frac{4\sigma^2_b}{b^2}\sin^2\frac{\alpha}{2}\cos^2\frac{\delta}{2}+ \\ +\Big[\Big(\frac{\sigma^2_v}{c^2}\cos^2\epsilon+\sigma^2_\epsilon\frac{v^2}{c^2}\sin^2\epsilon\Big)\sin^2\psi &+ \sigma^2_\psi\frac{v^2}{c^2}\cos^2\epsilon\cos^2\psi\Big]\sin^2\alpha\cos^4\frac{\delta}{2}. \qquad (17)\end{aligned}$$

For the wide-angle astrometric observations with FoR$\sim\frac{\pi}{12}$, the interferometer is expected to perform at the level of $\sigma_\alpha = 4\ \mu as$ and $\sigma_\delta \sim 1$ mas. Expression (17) suggests that for observations in the direction perpendicular to the baseline (e.g. when angle δ takes non-zero values), there will be a large contribution coming from σ_δ to the error budget, thus weakening the relativistic stellar aberration requirement. However, for the observations parallel to the baseline (e.q $\delta = 0$), influence of σ_δ is vanishes and one obtains the following equation:

$$\begin{aligned}\sigma^2_\alpha \cos^2\frac{\alpha}{2} &= \frac{\sigma^2_{\delta_{c_0}}}{b^2}+\frac{4\sigma^2_b}{b^2}\sin^2\frac{\alpha}{2}+ \\ &+ \Big[\Big(\frac{\sigma^2_v}{c^2}\cos^2\epsilon+\sigma^2_\epsilon\frac{v^2}{c^2}\sin^2\epsilon\Big)\sin^2\psi+\sigma^2_\psi\frac{v^2}{c^2}\cos^2\epsilon\cos^2\psi\Big]\sin^2\alpha. \qquad (18)\end{aligned}$$

This expression depends on the orientation of the spacecraft velocity with respect to the SSB coordinate reference frame. Let us analyze the worst case of the spacecraft orbital orientation. This orientation is when the spacecraft is moving towards/from the tile in the direction exactly normal to the tile of interest. In this case, when $\psi = \pm\frac{\pi}{2},\ \epsilon = 0$ (or in the direction normal to the tile), from the equation Eq.(18) one immediately obtains expression that contains a requirement on the error in the magnitude of the spacecraft's velocity:

$$\sigma^2_\alpha \cos^2\frac{\alpha}{2} = \frac{\sigma^2_{\delta_{c_0}}}{b^2}+\frac{4\sigma^2_b}{b^2}\sin^2\frac{\alpha}{2}+\frac{\sigma^2_v}{c^2}\sin^2\alpha. \qquad (19)$$

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[1] Note that this is not true for a general case of studying the stability of the reference grid (Swartz (1998)). Thus, one finds that the correlation in `RA` and `DEC` becomes a source for the zonal errors in the analysis of the grid accuracy and stability.

[2] A more general analysis should include a possible correlation between the constant term Δc_0 and the two angular components of velocity $\Delta\epsilon,\ \Delta\psi$. We will address this possibility later in the Appendix C and will discuss the components $\texttt{cov}(c_0,\epsilon)$ and $\texttt{cov}(c_0,\psi)$ of the total covariance matrix.

One could also verify, that errors in both sky angles ψ and ϵ are related to that for the velocity magnitude v and are given by

$$\sigma^2_\psi = \sigma^2_\epsilon = \frac{\sigma^2_v}{v^2}. \tag{20}$$

Note that a possible correlation between the constant term δc_0 and the two sky angles of velocity ψ, ϵ seriously affecting these estimates (a more detailed analysis of this problem is given in Appendix C). These errors, σ_ϵ and σ_ψ, are related to the error in the velocity magnitude σ_v as follows:

$$\begin{aligned}
\sigma_\epsilon &= \sqrt{\frac{\sigma^2_v}{v^2} + \Big[\frac{\sigma_{\delta_{c_0}}}{b}\frac{\rho(c_0,\epsilon)}{\sin\alpha}\,\frac{c}{v}\Big]^2} - \frac{\sigma_{\delta_{c_0}}}{b}\frac{\rho(c_0,\epsilon)}{\sin\alpha}\,\frac{c}{v} \ \geq\ 0, \\
\sigma_\psi &= \sqrt{\frac{\sigma^2_v}{v^2} + \Big[\frac{\sigma_{\delta_{c_0}}}{b}\frac{\rho(c_0,\psi)}{\sin\alpha}\,\frac{c}{v}\Big]^2} - \frac{\sigma_{\delta_{c_0}}}{b}\frac{\rho(c_0,\psi)}{\sin\alpha}\,\frac{c}{v} \ \geq\ 0,
\end{aligned} \tag{21}$$

where

$\sigma_\epsilon,\ \sigma_\psi$ – the errors in the two sky angles of the barycentric velocity vector $\vec{v}$;

$\rho(c_0,\epsilon),\ \rho(c_0,\psi)$ – correlation factors between the constant term and the two sky angles of the spacecraft's barycentric velocity, $|\rho(c_0,\epsilon)|, |\rho(c_0,\psi)| \leq 1$.

In addition to the velocity knowledge requirement one will have to impose two additional constraints on the quality of navigation data delivered by DSN. These two constraints are on the sampling rate Δt and positional accuracy of the spacecraft determination Δr together with the requirement on the stochastic acceleration control during the time of astrometric observations.

2.1.2. Sampling rate and orbital position knowledge

As we discussed previously, the correction for the annual stellar aberration may reach $\sim$ 20.5 arcsec. This allows estimation of the time interval on which this correction must be introduced in order to maintain the nominal accuracy. Thus, contribution of only the velocity of the spacecraft to the total error budget $\Delta\alpha$ may be obtained from Eqs.(16). For the motion in the plane of ecliptic with zero declination ($\delta = \epsilon = 0$), one will have:

$$\Delta\alpha\,\cos\frac{\alpha}{2} = \Big(\frac{\Delta v}{c}\sin\psi \ +\ \Delta\psi\ \frac{v}{c}\ \cos\psi\Big)\sin\alpha. \tag{22}$$

How often one will needs to introduce a correction for the stellar aberration? This question may be answered directly by analyzing the motion of the spacecraft in the plane

parallel to the tile, when the aberration takes it's largest value. The latitude argument for circular motion coincides with the mean anomaly, e.q. $\psi = n\,t$, where $n = \frac{2\pi}{P}$ and P is the period of orbital motion of the spacecraft. Taking this into account and setting $(\psi = 0; \epsilon = 0)$, from equation (22) one obtains:

$$\Big(\frac{d\Delta\alpha}{d\psi}\Big)\,\cos\frac{\alpha}{2}\,\Delta\psi = \frac{v}{c}\,\sin\alpha\,\frac{2\pi}{P}\Delta t, \tag{23}$$

where $\Delta\psi = n\,\Delta t$ and Δt is the time interval. After averaging, the last equation may be presented in a more useful form (note, $\sigma_\alpha \sim \overline{\frac{d\Delta\alpha}{d\psi}\,\Delta\psi}$), namely:

$$\sigma_\alpha\,\cos\frac{\alpha}{2} \geq \frac{v}{c}\,\sin\alpha\,\frac{2\pi}{P}\Delta t, \tag{24}$$

As a result, one will have to correct for the relativistic stellar aberration with such a sampling rate Δt that the expression on the right-hand side of the equation above will be considerably less then on the left side.

The data sampling rate Δt may be used to define the corresponding accuracy of determination of the spacecraft's position on it's orbit $\Delta r_{||}$. By decomposing the spacecraft's orbital position vector on the component parallel to velocity vector and the one perpendicular as given by $\Delta\vec{r} = \Delta\vec{r}_\perp + \Delta\vec{r}_{||}$, one may estimate the magnitudes for the both components involved. The expression for the magnitude of $\Delta\vec{r}_{||}$ may be obtained simply as follows:

$$\Delta r_{||} \leq v_{\mathtt{SIM}}\,\Delta t. \tag{25}$$

A requirement on the magnitude of the second component of the spacecraft position vector $\Delta\vec{r}_\perp$ may be derived from another mission goal, namely the expected accuracy for parallax determination. Thus, SIM is expected to achieve a mission accuracy of the determination of parallax at the level of $\sigma_\pi = 1$ μas for a nearby stars. This goal may be transformed into the requirement on the accuracy of the radial component of the barycentric spacecraft position $\Delta\vec{r}_\perp$. On may show that the error, introduced by annual parallax into the astrometric observations of a pair of stars separated on the sky by angle α (with $\delta = 0$), may be given as below:

$$\delta\alpha = \alpha_1 - \alpha_2 = \frac{r^{[\pi]}_\perp}{\mathcal{D}} 2\cos\Big(\psi - \frac{\alpha_1+\alpha_2}{2}\Big)\sin\frac{\alpha_2-\alpha_1}{2}, \tag{26}$$

where $r^{[\pi]}_\perp$ is the spacecraft barycentric distance (the superscript $[\pi]$ is used to separate parallactic distance requirement from those derived with the help of some other methods), and $\mathcal{D}$ is the distance to the object of study. Then for the tile in the SIM nominal observing

direction, defined by Eqs.(14), and for the motion perpendicular to the tile ($\psi = \pm\frac{\pi}{2}$) one may obtain following relation:

$$\Delta r_{\perp}^{[\pi]} \leq \sigma_\pi \, \mathcal{D} \frac{1}{2\sin\frac{\alpha}{2}}. \tag{27}$$

This relation suggests that the closer the distance $\mathcal{D}$ to the observed object the better should be the knowledge of the barycentric position of the spacecraft.

Accuracy of the spacecraft orbital position determination Δr is one of the navigational products that will be provided by means of DSN. However, one should not expect that DSN will provide such a continuous trucking of the spacecraft with the data sampling rate Δt. This constraint (primarily cost of such an extended DSN commitment) implies that a real-time on-board correction must be done in order to provide the conditions necessary for astrometric observations with expected accuracy. Note that this problem may be already solved by means of the SIM multiple-baseline architecture, when the two guide interferometers will take care of a number of similar effects. In any case, this question is out of scope of the present analysis, but it should be re-visited as a subject for separate study.

2.1.3. Acceleration knowledge constraint

The two additional constraints, namely on the quality of the DSN data indirectly imposed by the expected accuracy for proper motions determined for a number of stars during the mission's five-years live time, which will be of the order of $\mu_\alpha = 1$ μas/yr. Such a constraint may be also obtained from the Eq.(22). By differentiating this equation with respect to time one obtains:

$$\frac{d\Delta\alpha}{dt}\cos\frac{\alpha}{2} = \Big(\frac{1}{c}\frac{d\Delta v}{dt}\sin\psi \; + \; \frac{d\Delta\psi}{dt} \; \frac{v}{c} \; \cos\psi\Big)\sin\alpha. \tag{28}$$

The time derivatives on the right-hand side of this equations are the components of the spacecraft acceleration. Indeed, one may decompose vector of this acceleration onto the components normal to the orbit (radial) and the tangential one as follows $\vec{a} = \vec{a}_\perp + \vec{a}_{||}$, where $a_\perp = \frac{d\Delta v}{dt}$ and $a_{||} = \frac{d\Delta\psi}{dt}$. Assuming that the errors in both accelerations are normally distributed and uncorrelated, this equation may be averaged and presented in the following useful form:

$$\sigma^2_{\mu_\alpha}\cos^2\frac{\alpha}{2} = \Big[\frac{1}{c^2}\Big(\overline{\frac{d\Delta v}{dt}}\Big)^2\sin^2\psi \; + \; \Big(\overline{\frac{d\Delta\psi}{dt}}\Big)^2 \; \frac{v^2}{c^2} \; \cos^2\psi\Big]\sin^2\alpha. \tag{29}$$

For the motion of the spacecraft in the direction normal to the tile of interest ($\psi = \pm\frac{\pi}{2}$), one may obtain expression that imposes a requirements on the knowledge of the spacecraft

radial acceleration the following form:

$$\sigma_{\mu_\alpha}\,\cos\frac{\alpha}{2} \geq \frac{1}{c}\overline{\frac{d\Delta v}{dt}}\sin\alpha. \tag{30}$$

Similarly, for the motion of the spacecraft in the direction parallel to the tile ($\psi = 0$), one will have a requirement on the knowledge of the angular acceleration:

$$\sigma_{\mu_\alpha}\,\cos\frac{\alpha}{2} \geq \frac{v}{c}\overline{\frac{d\Delta\psi}{dt}}\sin\alpha. \tag{31}$$

These two equations suggest that, for accurate determination of proper motions one will need to control all the non-gravitational forces acting on the spacecraft at a very high level of accuracy. Note, for the expected orbital parameters of SIM, the control of the radial accelerations should be at the level of $\sim 1.2\times 10^{-13}$ km/s^2.

3. Relativistic stellar aberration requirements

In this Section we will present the estimates for a different strategies of accessing the influence of the relativistic stellar aberration on the expected accuracy of astrometric measurements with SIM.

We will begin from the expressions describing the uncorrelated data Eqs.(19)-(20). Remembering that angles α,δ vary in the range given by $\alpha^2+\delta^2 \leq (\frac{\pi}{12})^2$ for which $\cos\frac{\alpha}{2}$ never vanishes, one can divide the both sides of the equation (19) on $\cos^2\frac{\alpha}{2}$:

$$\sigma^2_\alpha \;=\; \frac{\sigma^2_{\delta_{c_0}}}{b^2\cos^2\frac{\alpha}{2}} + \frac{4\sigma^2_b}{b^2}\,\tan^2\frac{\alpha}{2} + \frac{4\sigma^2_v}{c^2}\,\sin^2\frac{\alpha}{2}. \tag{32}$$

The obtained equation represents ellipsoid with half-axis defined as follows:[3]

$$\sigma_{\delta_{c_0}} \;=\; \sigma_\alpha\, b\,\cos\frac{\alpha}{2}, \tag{33}$$

$$\sigma_b \;=\; \sigma_\alpha\,\frac{b}{2}\,\cot\frac{\alpha}{2}, \tag{34}$$

$$\sigma_v \;=\; \sigma_\alpha\,\frac{c}{2\sin\frac{\alpha}{2}}. \tag{35}$$

[3]A more general expression for the stellar aberration only may be obtained from Eqs.(11)-(13). It depends on the coordinates of both stars under consideration and has the following from: $\sigma_{\alpha_v} = \frac{\sigma_v}{c}\frac{\sin(\alpha_2-\alpha_1)}{\sin\alpha_2}$.

The necessary expressions for the tolerable errors in the components of the three-dimensional vector of the spacecraft's velocity may be obtained directly from Eqs.(20),(35) as

$$\sigma_\psi = \sigma_\epsilon = \sigma_\alpha \frac{c}{v_{\tt SIM}} \frac{1}{2\sin\frac{\alpha}{2}}. \tag{36}$$

In addition to these four constraints we will have to define the other three discussed in the previous Section 2.1, namely 1) the data sampling rate, Δt; 2) the accuracy of the orbital position determination, $\Delta r_\perp$ and $\Delta r_{||}$; and 3) the accuracy of compensation of the non-gravitational accelerations, $a_\perp = \overline{\frac{d\Delta v}{dt}}$ and $a_{||} = \overline{\frac{d\Delta\psi}{dt}}$. By utilizing equations (24)-(27) together with Eqs.(30),(31), these constraints may be presented by the following set of equations:

$$\Delta t \;\leq\; \sigma_\alpha \frac{c}{2\sin\frac{\alpha}{2}} \frac{P}{2\pi v_{\tt SIM}}, \tag{37}$$

$$\Delta r_{||} \leq \sigma_\alpha \frac{c}{2\sin\frac{\alpha}{2}} \frac{P}{2\pi}, \qquad \Delta r_\perp^{[\pi]} \leq \sigma_\pi \, \mathcal{D} \frac{1}{2\sin\frac{\alpha}{2}}, \tag{38}$$

$$\overline{\frac{d\Delta v}{dt}} \leq \sigma_{\mu_\alpha} \frac{c}{2\sin\frac{\alpha}{2}}, \qquad \overline{\frac{d\Delta\psi}{dt}} \leq \sigma_{\mu_\alpha} \frac{c}{v_{\tt SIM}} \frac{1}{2\sin\frac{\alpha}{2}}. \tag{39}$$

These five requirements are the derived constraints on the orbital motion of the spacecraft and the data sampling rate. While the stellar aberration constraint σ_v is the most important among those imposed by the orbital dynamics, we think that it is useful to have all the requirements related to the motion of the spacecraft spelled out.

Let us assume that a different error-budget constituents are contributing individually to the total error budget with an individual astrometric error $\sigma_\alpha = \Delta k \;\; \mu$as, where Δk is some number. By taking into account the numeric values for different quantities involved

$$\begin{aligned}
\sigma_\alpha &= \Delta k \;\; \mu\text{as}, & c \;&= 2.997292\times 10^{11} \text{ mm/s}, \\
b &= 10.50 \text{ m}, & 1\;\mu\text{as} \;&= 4.84814\times 10^{-12} \text{ rad}, \\
\sigma_{\mu_\alpha} &= \Delta l \;\; \mu\text{as/yr}, & \sigma_\pi \;&= \;\Delta q \;\; \mu\text{as}, \\
v_{\tt SIM} \approx v_\oplus \;&= 2.98\times 10^7 \;\; \text{mm/s}, & P \;&= 3.1536\times 10^7 \text{ s}
\end{aligned} \tag{40}$$

one could compute tolerable astrometric contributions introduced by the different errors budget constituents. The largest effect (the worst case) will be in the case of maximal separation between the two stars which will equal to the field of regard of the instrument $\alpha_{\tt max}$ =FoR$\equiv \frac{\pi}{12}$. Substituting the numerical quantities from (40) we immediately obtain the

following values for various components of the error budget:

$$\begin{aligned}
\sigma_{\delta_{c_0}} &= 50.470\ \Delta k \quad \text{pm}, \\
\sigma_b &= 193.332\ \Delta k \quad \text{pm}, \\
\sigma_v &= 5.566\ \Delta k \quad \text{mm/s}, \\
\sigma_\psi = \sigma_\epsilon &= 38.529\ \Delta k \quad \text{mas};
\end{aligned} \tag{41}$$

$$\Delta t = 0.938\ \Delta k \quad \text{s},$$

$$\begin{aligned}
\Delta r_{||} &= 27.952\ \ \Delta k \quad \text{km}, & \Delta r_\perp^{[\pi]} &= 573\ \ \Delta q \left(\frac{\mathcal{D}}{1\ \text{pc}}\right)\ \text{km}, \\
\overline{\frac{d\Delta v}{dt}} &= 11.132\ \Delta l \quad \text{mm/s/yr}, & \overline{\frac{d\Delta \psi}{dt}} &= 77.058\ \Delta l \quad \text{mas/yr},
\end{aligned} \tag{42}$$

where we have assumed that the barycentric velocity of the spacecraft is approximately equal to that of Earth' orbital motion around the sun, (e.g. $c/v_{\texttt{SIM}} = 10058$), thus giving us the requirement for the two sky angles of the velocity vector.

The constraints on the spacecraft's position are given by the Eqs.(42). It is worth noting that the parallactic requirement on the spacecraft barycentric position $\Delta r_\perp^{[\pi]}$ is easy to meet for a distant objects, because it grows linearly with distance. [Note that for the observations of the solar system objects one will have to use a completely different observational strategy.] The accuracies of the DSN navigation are much superior than it needed to satisfy this parallactic requirement. Note that the general relativistic deflection of light is also distance-dependent effect. One may think that this dependence may produce an independent constraint on $\Delta r_\perp^{[\pi]}$. However, a crude estimate shows that this is not the case and general relativity does not require a significant accuracy of knowledge of the barycentric distance of the spacecraft. This is why we will omit the constraint on $\Delta r_\perp^{[\pi]}$ from our studies. The last two constraints in Eqs.(42), on the two components of the spacecraft acceleration. These constraints are suggesting that the total uncompensated error in the acceleration of the craft over a half-orbit (half-year time interval) of the mission should not exceed these numbers. The first requirement may be re-written in more familiar units as $\overline{\frac{d\Delta v}{dt}} = 3.53 \times 10^{-13}\ \Delta l\ \ \text{km/s}^2$, which again implies quite a significant navigational involvement or a real-time on-board processing.

The worst case observation scenario will be realized for those angles ψ and ϵ which will maximize the corresponding multipliers in formulae (18). Thus, the motion of the spacecraft in the direction perpendicular to the tile, or $(\psi \to \psi_\perp = -\frac{\pi}{2},\ \epsilon = 0)$, will provide the most stringent requirement for the accuracy of knowledge of the magnitude of barycentric velocity of the spacecraft. Additionally, the motion in the direction parallel to the tile,

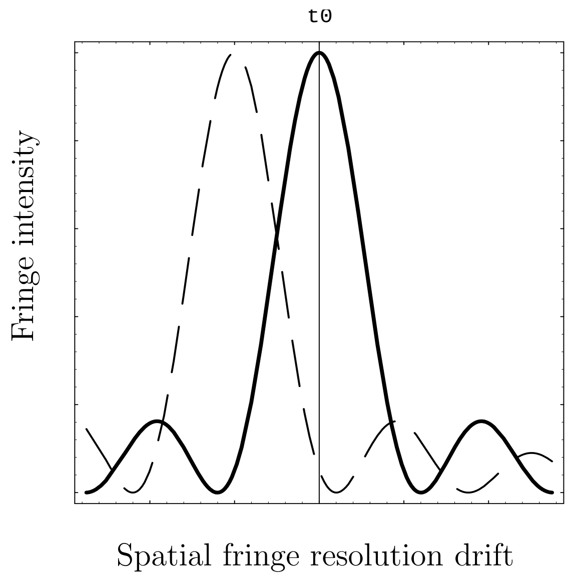

Fig. 4.— Spatial fringe drift introduced by the relativistic stellar aberration. Suppose the data are taken with the sampling rate Δt, then the thick curve – is the fringe intensity at the beginning of observations at time t_0, while the dotted curve is plotted for the position of the same fringe after time interval Δt has elapsed.

or $(\psi \to \psi_{||} = 0, \pi;\ \epsilon = 0)$, will provide the most stringent requirement for the accuracy of knowledge of the angular position of the spacecraft with respect to the solar system barycenter and, therefore, it introduces constraint on the data sampling rate Δt. This rate is necessary to apply for on-board navigation in order to correct for the stellar aberration. The nature of this correction for a single-baseline interferometer is presented by the Figure 4.

Relativistic stellar aberration introduces a spatial fringe drift due to the motion of the craft. This spatial fringe drift may be corrected if one will be sampling data with the rate Δt. This time interval tells how often one will need to introduce a correction to the velocity of a single-baseline interferometer in order to provide the necessary dynamical conditions for astrometric measurements with accuracy σ_α (i.e. for the starts separated by FoR=α). SIM is the multiple baseline interferometer. The main function of the two guide interferometers is to provide this sort of anchoring to the sky by providing a set of differential navigation parameters. Moreover, for the direction of motion parallel to the tile of interest, the accuracy of the angular position of the spacecraft is more important then it's velocity magnitude, thus

relaxing quite a bit the corresponding values for Δk in Eqs.(42). So, to the first order, the requirement imposed by Δt does not significantly impacting SIM astrometric campaign, but it is important enough to consider it for further studies.

3.1. Two ways of direct establishing the error allocations

In this Section we will present analysis of different methods for establishing the errors allocations for the stellar aberration. The expressions Eq.(41) are representing the set of requirements (including the stellar aberration) for the astrometric observations with accuracy of $\sigma_\alpha = \Delta k$ μas in the direction parallel to baseline. By varying the coefficient Δk we will generate the sets of requirements for $\sigma_{\delta_{c_0}}, \sigma_b, \sigma_v, \sigma_\psi$, and σ_ϵ that correspond to these different methods of error allocations.

There exist a number of different ways how to estimate contributions of different constituents to the total astrometric error budget. For a more accurate analysis, one should perform a numerical simulations in order to estimate the impact of those constituents on the astrometric grid performance. At this point we would like to analyze a number of possible limiting cases, in order to provide a taste of different flavors to this problem. To do this we will use the results presented by the set Eqs.(41). In particular, we will analyze the constraints that should be placed on different error budget constituents in the cases when one choose to make the assumptions based on either single measurement accuracy or the expected wide-angle mission accuracy.

3.1.1. Single observation accuracy requirements

SIM is being designed to be able to achieve astrometric accuracy of a single measurements of the order of $\sigma_\alpha = 8$ μas. This is first useful number to use, in order to derive the maximal stellar aberration error, tolerable during the tile observation. The corresponding estimate may be obtained by substituting the value $\Delta k = 8$ into Eqs.(33)-(35). As a result,

we obtain the following values for different terms in the equation Eq.(32):

$$
\begin{aligned}
\sigma^{[8]}_{\delta_{c_0}} &= 403.76 \quad \text{pm},\\
\sigma^{[8]}_{b} &= 1546.66 \quad \text{pm},\\
\sigma^{[8]}_{v} &= 44.53 \quad \text{mm/s},\\
\sigma^{[8]}_{\psi} = \sigma^{[8]}_{\epsilon} &= 308.23 \quad \text{mas};
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
\Delta t^{[8]} &= 7.50 \quad \text{s},\\
\Delta r^{[8]}_{||} &= 223.62 \quad \text{km},
\end{aligned}
$$

$$
\overline{\frac{d\Delta v}{dt}}^{[\Delta l=8]} = 44.53 \quad \text{mm/s/yr}, \qquad \overline{\frac{d\Delta \psi}{dt}}^{[\Delta l=8]} = 616.46 \quad \text{mas/yr} \tag{44}
$$

A reasonable improvement, for the numbers presented above, comes from the statement that a contribution of any component of the total error budget in the right-hand side of the equation (32) should not exceed 10% of the total variance a single accuracy of σ^2_α. This gives a different correction factor $\Delta k^{0.1}$ which is calculated to be $\Delta k^{0.1} = 8\sqrt{0.1} = 2.52982$. The resulting numbers for the contributions to the error budget in a single measurement mode are follows:

$$
\begin{aligned}
\sigma^{[2.5]}_{\delta_{c_0}} &= 127.68 \quad \text{pm},\\
\sigma^{[2.5]}_{b} &= 489.10 \quad \text{pm},\\
\sigma^{[2.5]}_{v} &= 14.08 \quad \text{mm/s},\\
\sigma^{[2.5]}_{\psi} = \sigma^{[2.5]}_{\epsilon} &= 97.47 \quad \text{mas};
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
\Delta t^{[2.5]} &= 2.37 \quad \text{s},\\
\Delta r^{[2.5]}_{||} &= 70.72 \quad \text{km},
\end{aligned}
$$

$$
\overline{\frac{d\Delta v}{dt}}^{[\Delta l=2.5]} = 14.08 \quad \text{mm/s/yr}, \qquad \overline{\frac{d\Delta \psi}{dt}}^{[\Delta l=2.5]} = 194.94 \quad \text{mas/yr} \tag{46}
$$

It is important to point out that the velocity aberration issue has two flavors to it, in a sense that it is influencing both the narrow angle observations and (by the procedure of the astrometric grid reduction) the wide angle ones. This mean that the error coming from the velocity aberration knowledge inside one single tile, will propagate to the total accuracy of the wide angle observations. This is why it is important to estimate the corresponding maximal tolerable errors based on the expected wide angle astrometric accuracy.

The set of requirements represents Eqs.(45),(46) an optimistic expectation on the accuracy of the velocity determination. Due to the reason that the astrometric accuracy will

be improving as mission progresses, this set of relativistic stellar aberration requirements will be easily met by the DSN navigation. However, the assumption based on the single measurement accuracy is over-optimistic and we need to consider the most driving cases presented by the expected wide-angle mission accuracy.

3.1.2. Mission accuracy requirements

The wide angle astrometric observations with SIM are expected to be with a mission accuracy of $\sigma_\alpha = 4$ μas. Using this number, one may derive a different set of requirements for SIM, which will be exactly twice smaller than the ones given by Eq.(43). Additionally, by applying a conservative 10% approach (described above), one may derive another set of estimates. The wide angle astrometric mode together with the conservative approach, gives for Δk the number $\Delta k^{0.1} = 4\sqrt{0.1} = 1.26491$. This value of the correction factor results in the following contributions to the error budget:

$$\begin{aligned}
\sigma^{[1.3]}_{\delta_{c_0}} &= \quad 63.84 \quad \text{pm}, && (47)\\
\sigma^{[1.3]}_{b} &= \quad 244.55 \quad \text{pm}, && (48)\\
\sigma^{[1.3]}_{v} &= \quad 7.04 \quad \text{mm/s}, && (49)\\
\sigma^{[1.3]}_{\psi} = \sigma^{[1.3]}_{\epsilon} &= \quad 48.74 \quad \text{mas}; && (50)
\end{aligned}$$

$$\begin{aligned}
\Delta t^{[1.3]} &= \quad 1.19 \quad \text{s}, && (51)\\
\Delta r^{[1.3]}_{||} &= \quad 35.36 \quad \text{km}, && (52)
\end{aligned}$$

$$\overline{\frac{d\Delta v}{dt}}^{[\Delta l=1.3]} = \quad 7.04 \quad \text{mm/s/yr}, \qquad \overline{\frac{d\Delta \psi}{dt}}^{[\Delta l=1.3]} = \quad 97.47 \quad \text{mas/yr}. \tag{53}$$

There is another important issue which hasn't been addressed yet. That is a possible correlation between the calibration term δ_{c_0} and the velocity sky-angles (ψ, ϵ) given by Eq.(21). To estimate the influence of this possible correlation, we assume that the two correlation coefficients for the two angles involved are equal $\rho(c_0, \psi) = \rho(c_0, \epsilon) = \rho_0$. Then, by using the expressions (21) together with the estimates Eqs.(33)-(35) one could obtain an improved expression in order to derive requirements on the accuracy of knowledge of the two sky-angles of the spacecraft's velocity:

$$\sigma_\epsilon = \sigma_\psi = \sigma_\alpha \frac{c}{v} \frac{1}{2\sin\frac{\alpha}{2}} \Big(\sqrt{1+\rho_0^2} - \rho_0\Big) = \sigma_v \Big(\sqrt{1+\rho_0^2} - \rho_0\Big) \geq 0. \tag{54}$$

It is important to point out, that for the worst case of highly correlated quantities (ψ, ϵ) and c_0, e.q. $\rho_0 \sim 1$ the result Eq.(50) will have to be further reduced by a factor of $\sqrt{2} - 1 = 0.4142$, thus tightening the requirements for the accuracy of knowledge of the two velocity sky-angles as $\sigma_\epsilon = \sigma_\psi \sim 20$ mas. [Note that this estimate is for the worst case of correlation, when $\rho \sim 1$. For the case, when $\rho_0 \sim 0.5$ this correction factor is almost twice relaxed, namely $(1 - 0.5^2)^{\frac{1}{2}} - 0.5^2 = 0.868$.] This example suggests that a possible correlation between the velocity components and the constant term in the narrow-angle observations may put an additional demand on the quality of the DSN data. Thus for the worst case scenario, DSN will have to deliver data for all three components of the velocity vector with considerably smaller errors, say

$$\sigma_v^{\tt corr} = 2.916 \quad \text{mm/s}, \tag{55}$$

$$\sigma_\psi^{\tt corr} = \sigma_\epsilon^{\tt corr} = 20.186 \quad \text{mas}. \tag{56}$$

The two numbers above are the very pessimistic estimates for the required accuracy of determination the spacecraft velocity vector and are given for the worst case of highly correlated data. We have obtained this requirement by using the number for the expected mission accuracy for wide-angle astrometric observations together with a conservative allocation[4] for different errors in the total astrometric error budget.

3.2. Currently adopted error allocations

Presently the analysis of not only the stellar aberration, but also a number of the other important issues is complicated by the fact that a realistic model for the spacecraft and the instrument is absent. This is why the currently adopted values in the error budget were chosen more or less intuitively. Moreover, these numbers are the same as they were for the Earth-orbiting mission study. Now, when SIM is designed to be placed on the Earth trailing SIRTF-like solar orbit, these numbers should be verified for a better justification. The reason for doing that is not only the need for justification of the number of mm/s (this number is the same for both orbits), but rather time and the level of DSN commitment, necessary to reach the desired accuracy of velocity determination.

Remember that the numbers in Eqs.(50) were obtained based on the assumption that the errors on the right-hand side of the Eq.(19) are forming an ellipsoid with half-axes given

[4]This estimate was derived from the statement that the contribution of any component of the error budget in the right side of the equation (32) should not exceed 10% of the total variance σ_α^2, with $\sigma_\alpha = 4$ μas.

by Eqs.(33)-(35). In reality, one will have to minimize each constituent of the total error budget in a such a way that in any given time the sum of the terms on the right-hand side of the equation (19) will not be larger then the expected variance σ^2_α. Currently, the error budget estimations allocates for the stellar aberration $\sigma_{\theta_v} = 36$ pm. The corresponding Δk is then estimated to be of order $\Delta k = 0.701$ [based on $\sigma_{\theta_v} \equiv (\sigma^2_{\alpha_v} + \sigma^2_{\delta_v})^{\frac{1}{2}} \approx \sigma_{\alpha_v} = b\frac{\sigma_v}{c} = 36$ pm and b=10.50 m]. The requirements now will have to be modified as follows:

$$\begin{aligned}
\sigma^{[0.7]}_{\delta_{c_0}} &= 35.38 \quad \text{pm}, && (57)\\
\sigma^{[0.7]}_{b} &= 135.53 \quad \text{pm}, && (58)\\
\sigma^{[0.7]}_{v} &= 3.90 \quad \text{mm/s}, && (59)\\
\sigma^{[0.7]}_{\psi} = \sigma^{[0.7]}_{\epsilon} &= 27.01 \quad \text{mas}; && (60)
\end{aligned}$$

$$\begin{aligned}
\Delta t^{[0.7]}_{||} &= 0.66 \quad \text{s}, && (61)\\
\Delta r^{[0.7]} &= 19.59 \quad \text{km}, && (62)
\end{aligned}$$

$$\overline{\frac{d\Delta v}{dt}}^{[\Delta l=0.7]} = 3.90 \quad \text{mm/s/yr}, \qquad \overline{\frac{d\Delta \psi}{dt}}^{[\Delta l=0.7]} = 54.02 \quad \text{mas/yr} \qquad (63)$$

Despite the fact that we have a reasonable gap between our estimates Eq.(49) and the best experimental guess given by Eq.(59), there some other factors that are necessary to consider. As we demonstrated previously, a possible correlation between the constant term and the two velocity sky-angles Eqn.(54) may completely eliminate this gap and further reduce the estimates presented in the Section 3.1.2 (for the worst case of highly correlated data). This fact minimizes a tolerable errors Eqs.(47)-(53), reducing those down to the values Eqs.(57)-(63).

Accounting for this correlation directly in the expressions above, leading to $\sigma_v = 1.616$ mm/s. However, remember that the contribution of stellar aberration to the total error budget was chosen to be $\sigma_{\theta_v} = 36$ pm. In order to see whether or not this number should be kept as a maximum tolerable stellar aberration error, this problem should be addressed with a formal numerical treatment. In the mean time, we recommend that σ_{θ_v} be reduced to, say ~ 25 pm.

A possible increase of the field of regard (FoR), which is currently being discussed, will also results in minimizing σ_v. In the Figure 5 we present the dependence of the maximal tolerable error in the velocity determination σ_v as a function of FoR. One can see that relativistic stellar aberration increases roughly as the tile angle squared. Thus, increasing the tile size from 15 to 20 degrees puts a tighter requirement on the knowledge of the spacecraft velocity which is already difficult because of the earth trailing orbit. Therefore a further study of this problem is granted.

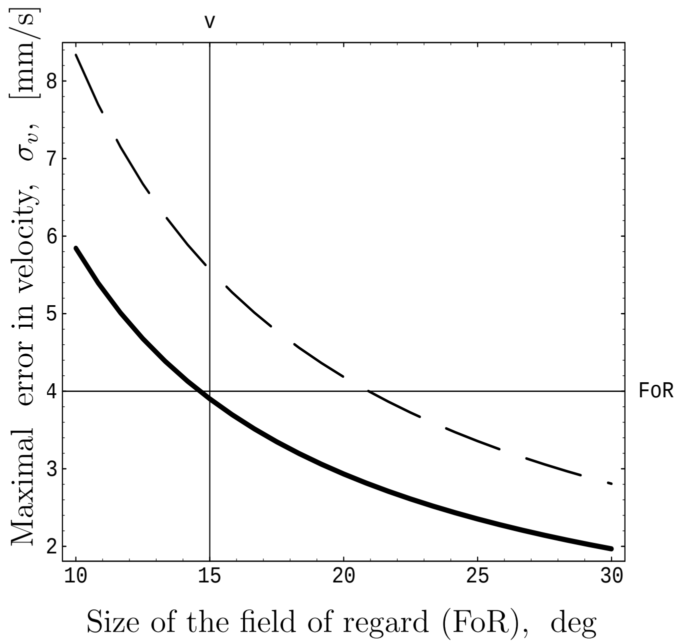

Fig. 5.— The size of the tolerable velocity error as a function of FoR. Plotted from Eq.(35) for the astrometric error of σ_α allocated for the relativistic stellar aberration in the total error budget. The upper dashed curve correspond to $\sigma_\alpha = 1$ μas and the lower thick curve is for $\sigma_\alpha = 0.701$ μas.

3.3. Requirements for on-board attitude control

In this Section we will estimate the influence of a temporal changes introduced by the rotational motion of the interferometer with respect to it's center of mass. Thus, the expression (50) may be used to derive another requirement, namely the requirement on the knowledge of the rotational motion of the spacecraft, or on-board attitude control. A simple-minded calculation assumes that, for example, the angle ψ is changing with time as

$$\psi(\tau) = \psi_0 + \dot{\psi}_0 \cdot \tau + \frac{1}{2}\ddot{\psi}_0 \cdot \tau^2 + \mathcal{O}(\tau^3), \tag{64}$$

where $\dot{\psi}_0$ and $\ddot{\psi}_0$ are the constant rate of this angular change and constant angular acceleration (a similar analysis could be made for the angle ϵ). Assuming that for any given time

the quantity $\psi(\tau)$ has a normal Gaussian distribution with errors obeying the equation:

$$\Delta\psi(\tau) = \Delta\dot{\psi}_0 \cdot \tau + \mathcal{O}(\tau^2). \tag{65}$$

the later expression may be taken as an input to derive the on-board attitude requirements. Let the time interval for sampling will be of the order of $\tau = \Delta t$ s then from Eqs.(41) and (65) one derives the following constraint:

$$\sigma_{\dot{\psi}_0} \sim 38.529 \cdot \frac{\Delta k}{\Delta t} \text{ mas/s.} \tag{66}$$

Thus, for the sampling rate of $\tau = 0.5$ s (e.q $\Delta t = 0.5$) and $\Delta k = 0.701$ (for absolute angular acceleration compensation, e.q. $\ddot{\psi}_0 = 0$), from this last equation (66) one obtains the following requirement on the accuracy of spacecraft attitude control:

$$\sigma^{[0.7]}_{\dot{\psi}_0} \sim \ 54.018 \text{ mas/s.} \tag{67}$$

Concluding, we would like to mention that the derived requirements Eqs.(43),(67) were based upon the assumption that the total error budget for the velocity aberration is $\sigma_{\alpha_v} = 0.701$ μas. Naturally, minimizing the error budget for the velocity aberration σ_{α_v} will result in tightening the derived constraints on the knowledge of the velocity vector itself $\vec{v} = v\Big(\cos\psi\cos\epsilon, \ \ \sin\psi\cos\epsilon, \ \ \sin\epsilon\Big)$ and the corresponding $\sigma_v, \sigma_\phi, \sigma_\epsilon$.

Conclusions

We have demonstrated that the 4 mm/s requirement for the relativistic stellar aberration is quite well justified for the SIM related studies. However, a secondary effects, such as possible correlation between the constant term and the instantaneous spacecraft velocity vector may produce a noticeable impact on the astrometric observations. Thus, account for this possible correlation, actually results in tightening the velocity requirement (for the worst case of highly correlated data) and almost twice minimizes a tolerable error in the velocity estimates, notably $\sigma_v = 1.616$ mm/s. Additionally, the solar radiation pressure will greatly influence the SIM orbital solution and will be the major force acting on the spacecraft. As a result the navigation DSN time will not go down as the mission progresses because of fluctuations in the solar radiation pressure.

These issues may be addressed by analyzing a number of different options that will present a grounds for possible trades between the Mission, Spacecraft, and Instrument Systems. Some of these options are 1). On-board processing of data, as oppose to increasing the use of DSN time; 2). The use of a precisely positioned solar shade for maintaining constant

pressure loading; 3). Velocity determination using integrated and time averaged accelerometers. In any case the development of a better model for the instrument in order to perform analysis of its 'elastic'/dynamical properties and the systematics hidden in the "constant" term will significantly boost the related studies.

Another issue that needed to be discussed is the reducing the velocity accuracy requirement down to 2 mm/s. A first look at this problem brings a conclusion is that this is not an easy problem for SIM in deep space (for more details, please see You and Ellis (1998); Gorham (1998)). Thus, some of the issues concerning SIM in a SIRTF-like orbit, with a 2mm/s velocity knowledge requirement are leading to the following conclusions from the DSN tracking stand-point:

1. By utilizing the Doppler range-rate method one can achieve more than adequate precision in the radial direction, of order 0.05 mm/s is reasonable. But in the plane of the sky, it is more difficult: long observations (doppler arcs) are necessary and it is unclear that the same precision can be obtained uniformly in any case – it will likely be a function of the ecliptic latitude of SIM at the time of observation.

2. By using the Delta-DOR observations (VLBI) with a single baseline and X/S-band observations, one may be able to obtain a precision of at best ~ 1 nrad for a fairly short observation (tens of minutes, including QSO calibrator). This corresponds to ~ 15 m at 0.1 AU. So, a pair of observations separated by ~ 2 hr or, so could get one plane-of-sky component to the required accuracy; ideally a track would be used to improve the SNR and get somewhat better accuracy perpendicular to the baseline. To get both plane of sky components a pair of baselines is necessary [a bigger DSN commitment, of course].

It is unclear whether the radial Doppler range-rate will be available with the same antennas at the same time as the VLBI observations. Development of a Ka band capability would improve the precision by a factor of ~ 3. This is why meeting the SIM requirements is not completely trivial. It might improve if the distance is not as far, and it would be interesting to see how performance varied with a number of parameters, including declination. There is another issue which was raised in this paper. That is a reasonable definition for the data sampling rate Δt necessary to use in order to introduce an on-board correction for the relativistic aberration. Our results show that the answer to this question may impact the whole Mission operations approach. This is why we believe that this issue should be a topic for a separate study and the corresponding results will be reported elsewhere.

Let us mention that the resulting SIM astrometric catalog will be derived by reducing the collected data to the solar system barycenter. In this case the uncertainty in Earth's barycentric velocity may be additional source of errors. Let us estimate the accuracy of the

presently known value for Earth' barycentric velocity $v_\oplus$. It is known that the Earth velocity uncertainty is dominated by the uncertainty in the Earth's orbital plane orientation, at the $\sim$1 mas level, giving a velocity uncertainty of $\sim$0.03 mm/s. However, there may be given another more conservative estimate to this quantity by using the accuracy of determination of the Earth-Moon barycenter, which is presently known to accuracy of about $\sim$ 1 km. This number helps to estimate the present accuracy of knowledge the Earth's barycentric velocity as $\sigma_{v_\oplus} = 0.2$ mm/s, thus offering a fair possibility for $\sim$ 1 μas astrometry [that, as we saw, requires $\sigma_v \sim$ 4mm/s]. So this is not a problem for SIM.

The requirements on the navigational accuracy obtained here, relate to the interferometer itself and not to the center of mass of the spacecraft. In the case of changing of the center mass of the spacecraft (which may occur due to different reasons, such as the rotation during imaging mode, or due to the use of propellant, etc.), the navigational parameters should be re-calculated to the required accuracy. Let us also mention that at the SIM level of accuracy a more subtle effects on the motion of the Earth about the sun will start to play an important role. These effects are the Moon's and the largest planets influence, and, probably, the body-body interaction between the Earth and the Moon, the spin-orbital connection in the Earth dynamics, the solar rotations and pulsations, influence of solar plasma, etc. All of these effects will have to be calculated and properly included into the models for astrometric grid simulations and corresponding data analysis. Our further studies will be aimed on improvement of the models used for the presented analysis by developing a better model for the instrument behavior and by expanding the parameter space to incorporate the other physical phenomena affecting astrometric observations from within the solar system.

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A. General expression for the errors propagation

In this Appendix we will present the intermediate calculations omitted for brevity in the main text. Parameterization Eq.(8)

$$
\begin{aligned}
\vec{s}_1 &= \big(\cos\alpha_1\cos\delta_1,\ \ \sin\alpha_1\cos\delta_1,\ \ \sin\delta_1\big),\\
\vec{s}_2 &= \big(\cos\alpha_2\cos\delta_2,\ \ \sin\alpha_2\cos\delta_2,\ \ \sin\delta_2\big),\\
\vec{v} &= v\big(\cos\psi\cos\epsilon,\ \ \sin\psi\cos\epsilon,\ \ \sin\epsilon\big),\\
\vec{b} &= b\big(\cos\mu\cos\nu,\ \ \sin\mu\cos\nu,\ \ \sin\nu\big).
\end{aligned}
\tag{A1}
$$

allows one to express the scalar products in Eq.(7) in the usual way:

$$
\begin{aligned}
(\vec{b}\cdot\vec{s}_1) &= b\Big(\cos\nu\cos\delta_1\cos(\alpha_1-\mu)+\sin\nu\sin\delta_1\Big),\\
(\vec{b}\cdot\vec{s}_2) &= b\Big(\cos\nu\cos\delta_2\cos(\alpha_2-\mu)+\sin\nu\sin\delta_2\Big),\\
(\vec{v}\cdot\vec{s}_1) &= v\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big),\\
(\vec{v}\cdot\vec{s}_2) &= v\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big),
\end{aligned}
\tag{A2}
$$

As a result, we can present equation (7) in the fully-blown form as follows:

$$
\begin{aligned}
\delta\ell &= b\Big[\cos\nu\Big(\cos\delta_1\cos(\alpha_1-\mu)-\cos\delta_2\cos(\alpha_2-\mu)\Big)+\sin\nu\Big(\sin\delta_1-\sin\delta_2\Big)\Big]+\delta c_0-\\
&- \frac{bv}{c}\Big[\Big(\cos\nu\cos\delta_1\cos(\alpha_1-\mu)+\sin\nu\sin\delta_1\Big)\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big)-\\
&- \Big(\cos\nu\cos\delta_2\cos(\alpha_2-\mu)+\sin\nu\sin\delta_2\Big)\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big)\Big].
\end{aligned}
\tag{A3}
$$

By taking the first derivative from both left and right sides of this equation (A8), we will obtain expression necessary to analyze the contributions of the different error factors involved in the problem under consideration to the overall error budget:

$$
\begin{aligned}
\Delta\delta\ell &= \sum_{i=1,2}\Big(\frac{\partial\delta\ell}{\partial\alpha_i}\Delta\alpha_i+\frac{\partial\delta\ell}{\partial\delta_i}\Delta\delta_i\Big)+\frac{\partial\delta\ell}{\partial b}\Delta b+\\
&+ \frac{\partial\delta\ell}{\partial\mu}\Delta\mu+\frac{\partial\delta\ell}{\partial\nu}\Delta\nu+\frac{\partial\delta\ell}{\partial v}\Delta v+\frac{\partial\delta\ell}{\partial\epsilon}\Delta\epsilon+\frac{\partial\delta\ell}{\partial\psi}\Delta\psi+\Delta\delta c_0.
\end{aligned}
\tag{A4}
$$

with the partial derivatives with respect to the observing angles δ_1,δ_2 and α_1,α_2 given as follows:

$$
\begin{aligned}
\frac{\partial \delta\ell}{\partial \delta_1} &= b\,\Big\{ -\cos\nu\sin\delta_1\cos(\alpha_1-\mu)+\sin\nu\cos\delta_1+ \\
&+\ \frac{v}{c}\Big[\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big)\Big(\cos\nu\sin\delta_1\cos(\alpha_1-\mu)-\sin\nu\cos\delta_1\Big)+ \\
&\quad +\Big(\cos\epsilon\sin\delta_1\cos(\alpha_1-\psi)-\sin\epsilon\cos\delta_1\Big)\Big(\cos\nu\cos\delta_1\cos(\alpha_1-\mu)+\sin\nu\sin\delta_1\Big)\Big]\Big\}, \\
\frac{\partial \delta\ell}{\partial \delta_2} &= b\,\Big\{ \cos\nu\sin\delta_2\cos(\alpha_2-\mu)-\sin\nu\cos\delta_2- \\
&-\ \frac{v}{c}\Big[\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big)\Big(\cos\nu\sin\delta_2\cos(\alpha_2-\mu)-\sin\nu\cos\delta_2\Big)+ \\
&\quad +\Big(\cos\epsilon\sin\delta_2\cos(\alpha_2-\psi)-\sin\epsilon\cos\delta_2\Big)\Big(\cos\nu\cos\delta_2\cos(\alpha_2-\mu)+\sin\nu\sin\delta_2\Big)\Big]\Big\}, \\
\frac{\partial \delta\ell}{\partial \alpha_1} &= b\,\Big\{ -\cos\nu\cos\delta_1\sin(\alpha_1-\mu)+ \\
&\quad +\ \frac{v}{c}\Big[\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big)\cos\nu\cos\delta_1\sin(\alpha_1-\mu)+ \\
&\quad\quad +\ \Big(\cos\nu\cos\delta_1\cos(\alpha_1-\mu)+\sin\nu\sin\delta_1\Big)\cos\epsilon\cos\delta_1\sin(\alpha_1-\psi)\Big]\Big\}, \\
\frac{\partial \delta\ell}{\partial \alpha_2} &= b\,\Big\{ \cos\nu\cos\delta_2\sin(\alpha_2-\mu)- \\
&\quad -\ \frac{v}{c}\Big[\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big)\cos\nu\cos\delta_2\sin(\alpha_2-\mu)+ \\
&\quad\quad +\ \Big(\cos\nu\cos\delta_2\cos(\alpha_2-\mu)+\sin\nu\sin\delta_2\Big)\cos\epsilon\cos\delta_2\sin(\alpha_2-\psi)\Big]\Big\},
\end{aligned}
\tag{A5}
$$

together with the partial derivatives with respect to components of the baseline vector $\vec{b}$:

$$
\begin{aligned}
\frac{\partial \delta\ell}{\partial b} &= \cos\nu\Big(\cos\delta_1\cos(\alpha_1-\mu)-\cos\delta_2\cos(\alpha_2-\mu)\Big)+\sin\nu\Big(\sin\delta_1-\sin\delta_2\Big)+\\
&+\frac{v}{c}\Big[\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big)\Big(\cos\nu\cos\delta_2\cos(\alpha_2-\mu)+\sin\nu\sin\delta_2\Big)-\\
&\quad-\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big)\Big(\cos\nu\cos\delta_1\cos(\alpha_1-\mu)+\sin\nu\sin\delta_1\Big)\Big],\\
\frac{\partial \delta\ell}{\partial \nu} &= b\,\Big\{\sin\nu\Big(\cos\delta_2\cos(\alpha_2-\mu)-\cos\delta_1\cos(\alpha_1-\mu)\Big)+\cos\nu\Big(\sin\delta_1-\sin\delta_2\Big)+\\
&+\frac{v}{c}\Big[\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big)\Big(-\sin\nu\cos\delta_2\cos(\alpha_2-\mu)+\cos\nu\sin\delta_2\Big)-\\
&\quad-\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big)\Big(-\sin\nu\cos\delta_1\cos(\alpha_1-\mu)+\cos\nu\sin\delta_1\Big)\Big]\Big\},\\
\frac{\partial \delta\ell}{\partial \mu} &= b\,\cos\nu\Big\{\cos\delta_1\cos(\alpha_1-\mu)-\cos\delta_2\cos(\alpha_2-\mu)+\\
&+\frac{v}{c}\Big[\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big)\cos\delta_2\sin(\alpha_2-\mu)-\\
&\quad-\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big)\cos\delta_1\sin(\alpha_1-\mu)\Big]\Big\},
\end{aligned}
\tag{A6}
$$

and, finally, with the following partials for the spacecraft's velocity:

$$
\begin{aligned}
\frac{\partial \delta\ell}{\partial v} &= \frac{b}{c}\Big[\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi)+\sin\epsilon\sin\delta_2\Big)\Big(\cos\nu\cos\delta_2\cos(\alpha_2-\mu)+\sin\nu\sin\delta_2\Big)-\\
&\quad-\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi)+\sin\epsilon\sin\delta_1\Big)\Big(\cos\nu\cos\delta_1\cos(\alpha_1-\mu)+\sin\nu\sin\delta_1\Big)\Big],\\
\frac{\partial \delta\ell}{\partial \epsilon} &= \frac{bv}{c}\Big[\Big(\sin\epsilon\cos\delta_1\cos(\alpha_1-\psi)-\cos\epsilon\sin\delta_1\Big)\Big(\cos\nu\cos\delta_1\cos(\alpha_1-\mu)+\sin\nu\sin\delta_1\Big)-\\
&\quad-\Big(\sin\epsilon\cos\delta_2\cos(\alpha_2-\psi)-\cos\epsilon\sin\delta_2\Big)\Big(\cos\nu\cos\delta_2\cos(\alpha_2-\mu)+\sin\nu\sin\delta_2\Big)\Big],\\
\frac{\partial \delta\ell}{\partial \psi} &= \frac{bv}{c}\cos\epsilon\Big[\Big(\cos\nu\cos\delta_2\cos(\alpha_2-\mu)+\sin\nu\sin\delta_2\Big)\cos\delta_2\sin(\alpha_2-\psi)-\\
&\quad-\Big(\cos\nu\cos\delta_1\cos(\alpha_1-\mu)+\sin\nu\sin\delta_1\Big)\cos\delta_1\sin(\alpha_1-\psi)\Big].
\end{aligned}
\tag{A7}
$$

The group of the expressions Eqs.(A4)-(A7) is quite difficult for analytical description, however it may be significantly simplified. First, without loosing generality one can set $\mu=\nu=0$, which equivalent to choosing the direction of the baseline vector $\vec{b}$ coinciding with x-axis, namely $\vec{b}=b(1,0,0)$. As a result of this choice, all vectors now will be counted

from the baseline and the expression (A3) for the relative OPD may be presented as follows:

$$\begin{aligned}\delta\ell &= b\Big(\cos\delta_1\cos\alpha_1 - \cos\delta_2\cos\alpha_2\Big) + \delta c_0 - \\ &- \frac{bv}{c}\Big[\cos\delta_1\cos\alpha_1\Big(\cos\epsilon\cos\delta_1\cos(\alpha_1-\psi) + \sin\epsilon\sin\delta_1\Big) - \\ &\qquad - \cos\delta_2\cos\alpha_2\Big(\cos\epsilon\cos\delta_2\cos(\alpha_2-\psi) + \sin\epsilon\sin\delta_2\Big)\Big]. \qquad \text{(A8)}\end{aligned}$$

Second, the largest expected ratio $\frac{v}{c} \approx \frac{v_\oplus}{c} = 9.942 \times 10^{-5}$, which makes the terms $\sim \frac{v}{c}$ in the expressions (A5) and (A6) of the second order of smallness. For the purposes of the present analysis, we may omit these terms and, after some re-arranging, expression Eq.(A4) may be presented as

$$\begin{aligned}\frac{\Delta\delta\ell}{b} &= \Delta\alpha_2\sin\alpha_2\cos\delta_2 - \Delta\alpha_1\sin\alpha_1\cos\delta_1 + \\ &+ \Delta\delta_2\cos\alpha_2\sin\delta_2 - \Delta\delta_1\cos\alpha_1\sin\delta_1 + \\ &+ \frac{\Delta b}{b}\Big[\cos\alpha_1\cos\delta_1 - \cos\alpha_2\cos\delta_2\Big] + \frac{\Delta\delta c_0}{b} + \\ &+ \frac{\Delta v}{c}\Big[\cos\epsilon\Big(\cos^2\delta_2\cos\alpha_2\cos(\alpha_2-\psi) - \cos^2\delta_1\cos\alpha_1\cos(\alpha_1-\psi)\Big) + \\ &\qquad + \sin\epsilon\Big(\cos\delta_2\sin\delta_2\cos\alpha_2 - \cos\delta_1\sin\delta_1\cos\alpha_1\Big)\Big] + \\ &+ \Delta\epsilon\,\frac{v}{c}\Big[\sin\epsilon\Big(\cos^2\delta_1\cos\alpha_1\cos(\alpha_1-\psi) - \cos^2\delta_2\cos\alpha_2\cos(\alpha_2-\psi)\Big) + \\ &\qquad + \cos\epsilon\Big(\cos\delta_2\sin\delta_2\cos\alpha_2 - \cos\delta_1\sin\delta_1\cos\alpha_1\Big)\Big] + \\ &+ \Delta\psi\,\frac{v}{c}\cos\epsilon\Big[\cos\alpha_2\cos^2\delta_2\sin(\alpha_2-\psi) - \cos\alpha_1\cos^2\delta_1\sin(\alpha_1-\psi)\Big]. \qquad \text{(A9)}\end{aligned}$$

This last expression may further be simplified by introducing a parameterization, which is natural for the problem under consideration. The final result for the first derivative of the

differential OPD may be given as follows:

$$
\begin{aligned}
\frac{\Delta\delta\ell}{b} &= \Delta\alpha \sin\alpha_2 \cos\delta_2 + \Delta\alpha_1 \Big(\sin\alpha_2 \cos\delta_2 - \sin\alpha_1 \cos\delta_1 \Big) + \\
&+ \Delta\delta \cos\alpha_2 \sin\delta_2 + \Delta\delta_1 \Big(\cos\alpha_2 \sin\delta_2 - \cos\alpha_1 \sin\delta_1 \Big) + \\
&+ \frac{\Delta b}{b} \Big(\cos\alpha_1 \cos\delta_1 - \cos\alpha_2 \cos\delta_2 \Big) + \frac{\Delta\delta c_0}{b} + \\
&+ \Big[\frac{\Delta v}{c} \cos(\epsilon - \epsilon_0) - \Delta\epsilon\ \frac{v}{c} \sin(\epsilon - \epsilon_0) \Big] \sqrt{(a^2 + f^2) \cos^2(\psi - \psi_0) + k^2} - \\
&- \Delta\psi\ \frac{v}{c} \cos\epsilon \sqrt{a^2 + f^2} \sin(\psi - \psi_0).
\end{aligned}
\tag{A10}
$$

where both angles ψ_0 and ϵ_0 are depend only on the positions of the two stars involved and are given as follows:

$$
\begin{aligned}
\sin\psi_0 &= \frac{f}{\sqrt{a^2 + f^2}}, \qquad \cos\psi_0 = \frac{a}{\sqrt{a^2 + f^2}}, \\
a &= \cos^2\delta_2 \cos^2\alpha_2 - \cos^2\delta_1 \cos^2\alpha_1, \\
f &= \cos^2\delta_2 \cos\alpha_2 \sin\alpha_2 - \cos^2\delta_1 \cos\alpha_1 \sin\alpha_1.
\end{aligned}
\tag{A11}
$$

$$
\begin{aligned}
\sin\epsilon_0 &= \frac{k}{\sqrt{(a^2 + f^2) \cos^2(\psi - \psi_0) + k^2}}, \\
\cos\epsilon_0 &= \frac{\sqrt{a^2 + f^2} \cos(\psi - \psi_0)}{\sqrt{(a^2 + f^2) \cos^2(\psi - \psi_0) + k^2}}, \\
k &= \cos\delta_2 \sin\delta_2 \cos\alpha_2 - \cos\delta_1 \sin\delta_1 \cos\alpha_1.
\end{aligned}
\tag{A12}
$$

B. Errors in the fully-parameterized relative OPD

Taking into account the fully parameterized form of the fractional relative OPD Eqs.(A4)-(A7) we may obtain the form of this quantity for a tile in the SIM nominal observing direction. Thus, substituting in these expressions the values for the positions of primary and secondary stars Eq.(14) we will have:

$$\begin{aligned}
\frac{\Delta\delta\ell}{b} \;&=\; \Delta\alpha\,\cos\nu\cos\frac{\delta}{2}\cos(\frac{\alpha}{2}-\mu)+2\Delta(\alpha_1-\mu)\cos\nu\cos\frac{\delta}{2}\sin\frac{\alpha}{2}\sin\mu\,- \\
&-\;\Delta\delta\Big[\cos\nu\sin\frac{\delta}{2}\sin(\frac{\alpha}{2}-\mu)\;+\;\sin\nu\cos\frac{\delta}{2}\Big]+2\Delta\delta_1\cos\nu\sin\frac{\delta}{2}\cos\frac{\alpha}{2}\sin\mu\,- \\
&-\;2\Delta\nu\Big[\sin\nu\cos\frac{\delta}{2}\sin\frac{\alpha}{2}\cos\mu+\cos\nu\sin\frac{\delta}{2}\Big]+ \\
&+\;\frac{2\Delta b}{b}\Big[\cos\nu\cos\frac{\delta}{2}\sin\frac{\alpha}{2}\cos\mu-\sin\nu\sin\frac{\delta}{2}\Big]\;+\;\frac{\Delta\delta c_0}{b}\;+ \\
&+\;\frac{\Delta v}{c}\Big[\cos\nu\Big(-\cos\epsilon\sin\alpha\cos^2\frac{\delta}{2}\,\sin(\psi+\mu)+\sin\epsilon\cos\frac{\alpha}{2}\sin\delta\sin\mu\Big)+ \\
&\qquad+\sin\nu\,\cos\epsilon\cos\frac{\alpha}{2}\sin\delta\sin\psi\Big]+ \\
&+\;\Delta\epsilon\,\frac{v}{c}\Big[\cos\nu\Big(\sin\epsilon\sin\alpha\cos^2\frac{\delta}{2}\,\sin(\psi+\mu)+\cos\epsilon\cos\frac{\alpha}{2}\sin\delta\sin\mu\Big)- \\
&\qquad-\sin\nu\,\sin\epsilon\cos\frac{\alpha}{2}\sin\delta\sin\psi\Big]+ \\
&+\;\Delta\psi\,\frac{v}{c}\cos\epsilon\,\Big(\sin\nu\,\cos\frac{\alpha}{2}\sin\delta\cos\psi-\cos\nu\sin\alpha\cos^2\frac{\delta}{2}\,\cos(\psi+\mu)\Big).
\end{aligned} \tag{B1}$$

One may see that the angles of the baseline orientation (μ,ν) are significantly influencing the narrow-angle astrometric observations. However, as previously we will choose both angles as $\mu=\nu=0$, which is equivalent to choosing the direction of the baseline vector $\vec{b}$ to coincide with x-axis, namely $\vec{b}=b(1,0,0)$. As a result of this choice, all vectors now will be counted from the baseline (see Figure 2). Resulted expression for the relative OPD may be presented in a simpler form, namely:

$$\begin{aligned}
\frac{\Delta\delta\ell}{b} \;&=\; \Delta\alpha\,\cos\frac{\alpha}{2}\cos\frac{\delta}{2}\;-\;\Delta\delta\,\sin\frac{\alpha}{2}\sin\frac{\delta}{2}\;+\;\frac{2\Delta b}{b}\,\sin\frac{\alpha}{2}\cos\frac{\delta}{2}\;+\;\frac{\Delta\delta c_0}{b}\;- \\
&-\;\Big[\Big(\frac{\Delta v}{c}\,\cos\epsilon\;-\;\Delta\epsilon\,\frac{v}{c}\,\sin\epsilon\Big)\sin\psi\;+\;\Delta\psi\,\frac{v}{c}\,\cos\epsilon\cos\psi\Big]\sin\alpha\cos^2\frac{\delta}{2}.
\end{aligned} \tag{B2}$$

Note that obtained expression does not depend on the errors in position of the primary star $\Delta\alpha_1, \Delta\delta_1$. The obtained result may now be used to analyze the propagation of errors in the future astrometric observations with SIM.

C. Components of the covariance matrix

The first term in the expression (B2) $\Delta\delta\ell$ may equivalently be presented as $\Delta\delta\ell = \Delta(n\lambda_0) = \Delta n\,\lambda_0 + n\,\Delta\lambda_0$, where λ_0 is the operating frequency and n is an integer number. This term vanishes because both λ_0 and n are assumed to be known to a sufficiently high accuracy, e.q. $\Delta n = 0$, $\Delta\lambda_0 = 0$. This is true due to the fact that the SIM white light fringe tracker will make sure that variations in these two quantities will be exactly zero. Then the remaining differentials Δv, $\Delta\epsilon$, $\Delta\psi$, $\Delta\alpha$, $\Delta\delta$, Δb and $\Delta\delta c_0$ will have to satisfy the equation:

$$\Delta\alpha\,\cos\frac{\alpha}{2}\cos\frac{\delta}{2} \;-\; \Delta\delta\,\sin\frac{\alpha}{2}\sin\frac{\delta}{2} \;=\; -\frac{\Delta\delta c_0}{b} - \frac{2\Delta b}{b}\,\sin\frac{\alpha}{2}\cos\frac{\delta}{2} + \\ +\Big[\Big(\frac{\Delta v}{c}\,\cos\epsilon \;-\; \Delta\epsilon\,\frac{v}{c}\,\sin\epsilon\Big)\sin\psi \;+\; \Delta\psi\,\frac{v}{c}\,\cos\epsilon\cos\psi\Big]\sin\alpha\cos^2\frac{\delta}{2}. \qquad \text{(C1)}$$

In order to simplify further analysis we have separated the terms in the expression above in a such a way, so that the left side of this equation represents the error in the measurement of the absolute angular separation between the two stars, while the left side shows the main contributing factors to this quantity.

In order to study the error propagation in the astrometric observations, we need to take the square of the expression (C1):

$$(\Delta\alpha)^2\,\cos^2\frac{\alpha}{2}\cos^2\frac{\delta}{2} + (\Delta\delta)^2\,\sin^2\frac{\alpha}{2}\sin^2\frac{\delta}{2} - \frac{(\Delta\alpha\Delta\delta)}{2}\,\sin\alpha\sin\delta \;= \\ = \frac{(\Delta\delta c_0)^2}{b^2} \;+\; \frac{4(\Delta b)^2}{b^2}\,\sin^2\frac{\alpha}{2}\,\cos^2\frac{\delta}{2} \;+\; 4\frac{(\Delta\delta c_0\Delta b)}{b^2}\,\sin\frac{\alpha}{2}\cos\frac{\delta}{2} + \\ +\;\Big[\Big(\frac{\Delta v}{c}\,\cos\epsilon \;-\; \Delta\epsilon\,\frac{v}{c}\,\sin\epsilon\Big)\sin\psi \;+\; \Delta\psi\,\frac{v}{c}\,\cos\epsilon\cos\psi\Big]^2\sin^2\alpha\cos^4\frac{\delta}{2} - \\ -\;\frac{2\Delta\delta c_0}{b}\Big[\Big(\frac{\Delta v}{c}\,\cos\epsilon \;-\; \Delta\epsilon\,\frac{v}{c}\,\sin\epsilon\Big)\sin\psi \;+\; \Delta\psi\,\frac{v}{c}\,\cos\epsilon\cos\psi\Big]\sin\alpha\cos^2\frac{\delta}{2} - \\ -\;\frac{4\Delta b}{b}\Big[\Big(\frac{\Delta v}{c}\,\cos\epsilon \;-\; \Delta\epsilon\,\frac{v}{c}\,\sin\epsilon\Big)\sin\psi \;+\; \Delta\psi\,\frac{v}{c}\,\cos\epsilon\cos\psi\Big]\sin\alpha\sin\frac{\alpha}{2}\cos^3\frac{\delta}{2}. \qquad \text{(C2)}$$

One may expect that in any given tile the errors in $\Delta\alpha$ and $\Delta\delta$ are normally distributed and uncorrelated.[5] The errors due to the orbital dynamics Δv, $\Delta\epsilon$, $\Delta\psi$ at the chosen

[5] Note that this is not true for a general case of studying the reference grid stability. Thus one finds that the correlation in `RA` and `DEC` becomes a source for the zonal errors in the analysis of the grid accuracy.

approximation may also be treated as being not correlated with instrumental errors $\Delta b,\ \Delta c_0$. As a result, the components of the covariance matrix may be given as follows:

$$
\begin{aligned}
\overline{(\Delta\alpha)^2} &= \sigma^2_\alpha, & \overline{(\Delta\delta)^2} &= \sigma^2_\delta, & \overline{\Delta\alpha\Delta\delta} &= 0,\\
\overline{(\Delta v)^2} &= \sigma^2_v, & \overline{(\Delta\epsilon)^2} &= \sigma^2_\epsilon, & \overline{(\Delta\psi)^2} &= \sigma^2_\psi,\\
\overline{\Delta v\Delta\epsilon} &= 0, & \overline{\Delta v\Delta\psi} &= 0, & \overline{\Delta\epsilon\Delta\psi} &= 0,\\
\overline{(\Delta\delta c_0)^2} &= \sigma^2_{\delta_{c_0}}, & \overline{\Delta\delta_{c_0}\Delta v} &= 0, & \overline{(\Delta b)^2} &= \sigma^2_b,\\
\overline{\Delta b\Delta v} &= 0, & \overline{\Delta b\Delta\epsilon} &= 0, & \overline{\Delta b\Delta\psi} &= 0.
\end{aligned}
\tag{C3}
$$

$$
\overline{\Delta\delta c_0\Delta b} = \sigma_{\delta_{c_0}}\sigma_b\,\rho(c_0,b), \qquad \overline{\Delta\delta c_0\Delta\epsilon} = \sigma_{\delta_{c_0}}\sigma_\epsilon\,\rho(c_0,\epsilon), \qquad \overline{\Delta\delta c_0\Delta\psi} = \sigma_{\delta_{c_0}}\sigma_\psi\,\rho(c_0,\psi).
\tag{C4}
$$

These expressions are helpful to present the result of averaging the equation (C2) in the following form:

$$
\begin{aligned}
&\sigma^2_\alpha\,\cos^2\frac{\alpha}{2}\cos^2\frac{\delta}{2} + \sigma^2_\delta\,\sin^2\frac{\alpha}{2}\sin^2\frac{\delta}{2} =\\
&\quad= \frac{\sigma^2_{\delta_{c_0}}}{b^2} + \frac{4\,\sigma^2_b}{b^2}\,\sin^2\frac{\alpha}{2}\,\cos^2\frac{\delta}{2} + \frac{4\,\sigma_{\delta_{c_0}}\sigma_b}{b^2}\rho(c_0,b)\,\sin\frac{\alpha}{2}\cos\frac{\delta}{2}\ +\\
&\quad+\ \Big[\Big(\frac{\sigma^2_v}{c^2}\,\cos^2\epsilon + \sigma^2_\epsilon\,\frac{v^2}{c^2}\,\sin^2\epsilon\Big)\sin^2\psi + \sigma^2_\psi\,\frac{v^2}{c^2}\,\cos^2\epsilon\,\cos^2\psi\Big]\sin^2\alpha\cos^4\frac{\delta}{2} +\\
&\quad+\ \frac{2\,\sigma_{\delta_{c_0}}}{b}\Big[\sigma_\epsilon\,\rho(c_0,\epsilon)\,\sin\epsilon\sin\psi\ -\ \sigma_\psi\,\rho(c_0,\psi)\,\cos\epsilon\cos\psi\Big]\,\frac{v}{c}\,\sin\alpha\cos^2\frac{\delta}{2}.
\end{aligned}
\tag{C5}
$$

Expression (C5) suggests that for the observations in the direction perpendicular to the baseline (e.q. when δ are non-zero), there will be a large contribution to the error budget coming from σ_δ. However, for observations parallel to the baseline (e.q $\delta = 0$) the influence of σ_δ is vanishes and one obtains the most stringent requirement on the velocity aberration. Then, by taking $\delta = 0$ one may present Eq.(C5) in the following form:

$$\sigma_\alpha^2\,\cos^2\frac{\alpha}{2} = \frac{\sigma^2_{\delta_{c_0}}}{b^2} + \frac{4\,\sigma_b^2}{b^2}\,\sin^2\frac{\alpha}{2} + \frac{4\,\sigma_{\delta_{c_0}}\sigma_b}{b^2}\rho(c_0,b)\,\sin\frac{\alpha}{2}\; +$$
$$+\;\Big[\Big(\frac{\sigma_v^2}{c^2}\,\cos^2\epsilon + \sigma_\epsilon^2\,\frac{v^2}{c^2}\,\sin^2\epsilon\Big)\sin^2\psi + \sigma_\psi^2\,\frac{v^2}{c^2}\,\cos^2\epsilon\,\cos^2\psi\Big]\sin^2\alpha\; +$$
$$+\;\frac{2\,\sigma_{\delta_{c_0}}}{b}\Big[\sigma_\epsilon\,\rho(c_0,\epsilon)\,\sin\epsilon\sin\psi\; - \;\sigma_\psi\,\rho(c_0,\psi)\,\cos\epsilon\cos\psi\Big]\,\frac{v}{c}\,\sin\alpha. \tag{C6}$$

Remembering further that angles (α,δ) vary in the range given by $\alpha^2+\delta^2\le(\frac{\pi}{12})^2$ and $\cos\frac{\alpha}{2}$ never vanishes in this interval, we can divide the both sides of the equation (C6) on $\cos^2\frac{\alpha}{2}$:

$$\sigma_\alpha^2 = \frac{\sigma^2_{\delta_{c_0}}}{b^2\,\cos^2\frac{\alpha}{2}} + \frac{4\,\sigma_b^2}{b^2}\,\tan^2\frac{\alpha}{2} + \frac{4\,\sigma_{\delta_{c_0}}\sigma_b}{b^2}\rho(c_0,b)\,\frac{\sin\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}\; +$$
$$+\;4\Big[\Big(\frac{\sigma_v^2}{c^2}\,\cos^2\epsilon + \sigma_\epsilon^2\,\frac{v^2}{c^2}\,\sin^2\epsilon\Big)\sin^2\psi + \sigma_\psi^2\,\frac{v^2}{c^2}\,\cos^2\epsilon\,\cos^2\psi\Big]\,\sin^2\frac{\alpha}{2}\; +$$
$$+\;\frac{4\,\sigma_{\delta_{c_0}}}{b}\Big[\sigma_\epsilon\,\rho(c_0,\epsilon)\,\sin\epsilon\sin\psi\; - \;\sigma_\psi\,\rho(c_0,\psi)\,\cos\epsilon\cos\psi\Big]\,\frac{v}{c}\,\tan\frac{\alpha}{2}. \tag{C7}$$

This equation represents an ellipsoid of with half-axis defined by the direction of motion of the spacecraft. Thus, for the spacecraft motion in the direction given by $\psi=\frac{\pi}{2}$ and $\epsilon=0$ we can have the first expression for astrometric errors:

$$\sigma_\alpha^2 = \frac{\sigma^2_{\delta_{c_0}}}{b^2\,\cos^2\frac{\alpha}{2}} + \frac{4\,\sigma_b^2}{b^2}\,\tan^2\frac{\alpha}{2} + \frac{4\,\sigma_{\delta_{c_0}}\sigma_b}{b^2}\rho(c_0,b)\,\frac{\sin\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}\; + \;\frac{4\,\sigma_v^2}{c^2}\,\sin^2\frac{\alpha}{2}. \tag{C8}$$

For the case of motion in the direction $\psi=\frac{\pi}{2}$ and $\epsilon=\pm\frac{\pi}{2}$ we have the second relation:

$$\sigma_\alpha^2 = \frac{\sigma^2_{\delta_{c_0}}}{b^2\,\cos^2\frac{\alpha}{2}} \; + \; \frac{4\,\sigma_b^2}{b^2}\,\tan^2\frac{\alpha}{2} \; + \; \frac{4\,\sigma_{\delta_{c_0}}\sigma_b}{b^2}\rho(c_0,b)\,\frac{\sin\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}\; +$$
$$+\;4\,\sigma_\epsilon^2\,\frac{v^2}{c^2}\,\sin^2\frac{\alpha}{2}\;\pm\;\frac{4\,\sigma_{\delta_{c_0}}}{b}\sigma_\epsilon\,\frac{v}{c}\,\rho(c_0,\epsilon)\,\tan\frac{\alpha}{2}. \tag{C9}$$

And, finally, the last expression may be obtained when considering the motion in the direction $\psi=0$ and $\epsilon=0,\,\pi$. This last constraint reads:

$$\sigma_\alpha^2 = \frac{\sigma^2_{\delta_{c_0}}}{b^2\,\cos^2\frac{\alpha}{2}} + \frac{4\,\sigma_b^2}{b^2}\,\tan^2\frac{\alpha}{2} \; + \; \frac{4\,\sigma_{\delta_{c_0}}\sigma_b}{b^2}\rho(c_0,b)\,\frac{\sin\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}\; +$$
$$+\;4\,\sigma_\psi^2\,\frac{v^2}{c^2}\,\sin^2\frac{\alpha}{2}\;\mp\;\frac{4\,\sigma_{\delta_{c_0}}}{b}\sigma_\psi\,\frac{v}{c}\,\rho(c_0,\psi)\,\tan\frac{\alpha}{2}. \tag{C10}$$

For the simplicity, we can relate the errors in the sky angles of velocity σ_ϵ and σ_ψ with that of the velocity magnitude σ_v. To obtain the most stringent constraints on the sky angle we will use to following expressions:

$$\begin{aligned}\frac{\sigma_v^2}{c^2}\sin^2\frac{\alpha}{2} &= \sigma_\epsilon^2\,\frac{v^2}{c^2}\sin^2\frac{\alpha}{2} \;+\; \sigma_\epsilon\,\frac{v}{c}\,\frac{\sigma_{\delta_{c_0}}}{b}\,\rho(c_0,\epsilon)\,\tan\frac{\alpha}{2} \;= \\ &= \sigma_\psi^2\,\frac{v^2}{c^2}\sin^2\frac{\alpha}{2} \;+\; \sigma_\psi\,\frac{v}{c}\,\frac{\sigma_{\delta_{c_0}}}{b}\,\rho(c_0,\psi)\,\tan\frac{\alpha}{2}.\end{aligned} \tag{C11}$$

As a result, in our further analysis we will concentrate on the equation Eq.(C8) together with the following two solutions for σ_ϵ and σ_ψ (which were obtained directly from Eqs.(C11)):

$$\begin{aligned}\sigma_\epsilon &= \sqrt{\frac{\sigma_v^2}{v^2}+\Big[\frac{\sigma_{\delta_{c_0}}}{b}\,\frac{\rho(c_0,\epsilon)}{\sin\alpha}\,\frac{c}{v}\Big]^2}-\frac{\sigma_{\delta_{c_0}}}{b}\,\frac{\rho(c_0,\epsilon)}{\sin\alpha}\,\frac{c}{v}\;\geq\;0, \\ \sigma_\psi &= \sqrt{\frac{\sigma_v^2}{v^2}+\Big[\frac{\sigma_{\delta_{c_0}}}{b}\,\frac{\rho(c_0,\psi)}{\sin\alpha}\,\frac{c}{v}\Big]^2}-\frac{\sigma_{\delta_{c_0}}}{b}\,\frac{\rho(c_0,\psi)}{\sin\alpha}\,\frac{c}{v}\;\geq\;0.\end{aligned} \tag{C12}$$

Just for the estimation purposes we can assume that $\rho(c_0,\psi)=\rho(c_0,\epsilon)=\rho_0$. Then, by using the expressions (C12) together with the estimates Eqs.(35) one could obtain improved requirements on the accuracy of knowledge of the two sky-angles of the spacecraft velocity:

$$\sigma_\epsilon=\sigma_\psi=\sigma_\alpha\,\frac{c}{v}\,\frac{1}{2\sin\frac{\alpha}{2}}\Big(\sqrt{1+\rho_0^2}-\rho_0\Big)\geq 0. \tag{C13}$$

Note, that for the worst case of highly correlated quantities (ψ,ϵ) and c_0, e.g. $\rho_0\sim 1$ the result Eq.(43) will have to be further reduced by a factor $\sqrt{2}-1=0.4142$, thus tightening the requirements for the accuracy of knowledge of the two velocity sky-angles as $\sigma_\epsilon=\sigma_\psi=20.186$ mas. This example suggests that a possible correlation between the velocity components and the constant term may be reduced if the DSN will deliver the data for all three components with a twice higher accuracy then planned.